MATH 1241
Common Final Examination
Multiple Choice Section
Fall 2001

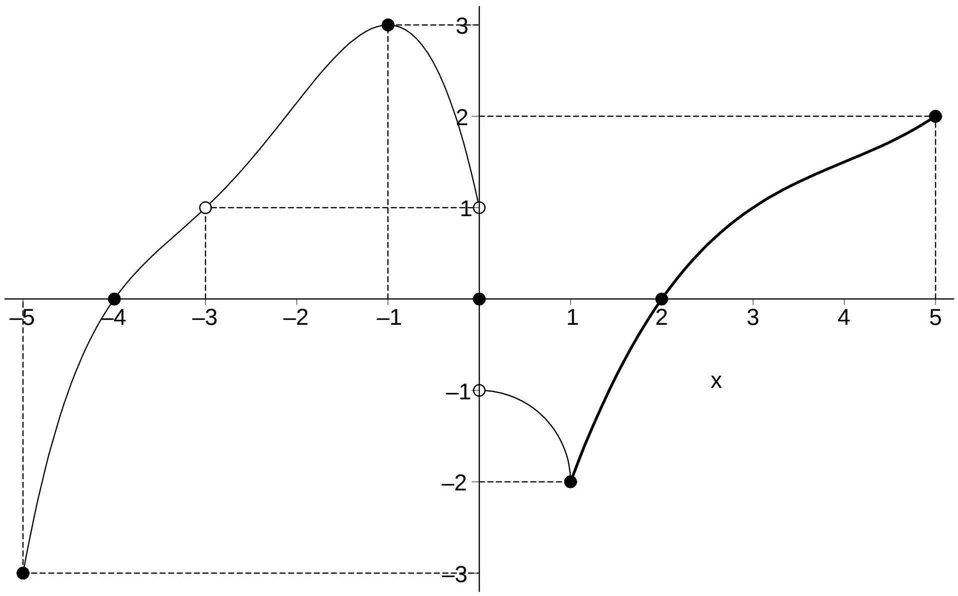
This exam is divided into two parts. These pages contain Part I which consists of 25 multiple choice questions. Part II consists of 6 free response questions. You have three hours for the entire test. Part I should be completed in two hours. Part II should be completed in one hour.

This part of the exam consists of 25 multiple choice questions. They are printed on the front and back of each page. Be sure that you answer 25 different questions. A special answer sheet is provided so that your answers can be machine graded.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**
- **The use of a TI-89 or TI-92 calculator on this test is a violation of the Code of Student Conduct**.

At the end of the examination you MUST hand in this test booklet, your answer sheet and all scratch paper.

Questions 1–4 refer to the function $f(x)$ whose graph is indicated below.



1. Select the true statement.

   a) $\lim_{x \to -3} f(x)$ does not exist.

   b) $\lim_{x \to -3} f(x) = 1$.

   c) $\lim_{x \to 0} f(x) = 1$.

   d) $\lim_{x \to 0} f(x) = 0$.

   e) $\lim_{x \to -3} f(x) = \lim_{x \to 0} f(x)$.

2. The value of $\lim_{x \to 0^+} f(x)$ is

   a) -1

   b) 0

   c) 1

   d) 2

   e) 4

3. The set on which the function $f(x)$ is increasing is the interval or union of intervals

   a) $(-5,-1)\cup(1,5)$

   b) $(-4,0)\cup(2,5)$

   c) $(-3,2)\cup(-2,3)$

   d) $(-2,1)$

   e) $(-4,2)\cup(2,5)$

4. The set of x-values at which the function $f(x)$ is continuous but *not* differentiable is

   a) $x=-3$

   b) $x=-2$

   c) $x=0$

   d) $x=1$

   e) $x=-2$ and $x=1$

5. Let f be a function such that

$$\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}=-3\,.$$

   Which of the following statements *must* be true?

   I. f is continuous at $x=4$
   II. f is differentiable at $x=4$
   III. f' is continuous at $x=4$

   a) Only I

   b) Only II

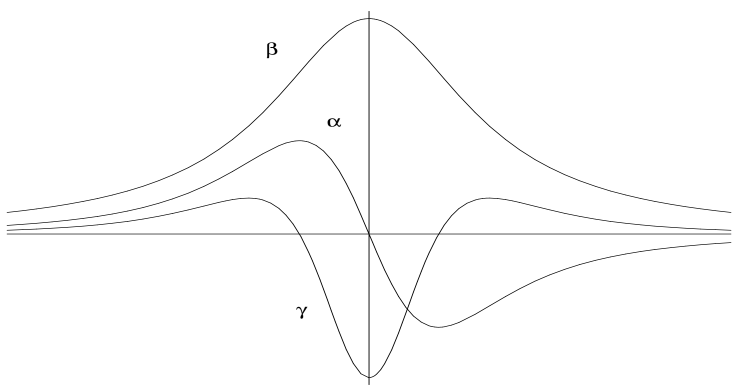
   c) Only III

   d) Both I and II, but not necessarily III

   e) Both II and III, but not necessarily I

6. If $f(x) = 2x^2 + 1$ and $g(x) = \sqrt{x-2}$, then $(f \circ g)(x)$ is
   a) $\sqrt{2x^2 - 1}$
   b) $(2x^2 + 1)\sqrt{x-2}$
   c) $2x - 3$
   d) $\sqrt{2x^2 - 1}$
   e) $2x^2 - 1$

7. One of the curves α, β, and γ is the graph of a function f, another is the graph of f', and the third is the graph of f''. Choose the correct identification.



   a) $(f, f', f'') = (\alpha, \beta, \gamma)$
   b) $(f, f', f'') = (\alpha, \gamma, \beta)$
   c) $(f, f', f'') = (\beta, \alpha, \gamma)$
   d) $(f, f', f'') = (\beta, \gamma, \alpha)$
   e) $(f, f', f'') = (\gamma, \alpha, \beta)$

8. If $f(x) = \sqrt{x^2+25}$, then $f'(x)$ is

a) 1

b) $\dfrac{1}{2\sqrt{x^2+25}}$

c) $\sqrt{2x}$

d) $\frac{2}{3}(x^2+25)^{\frac{3}{2}}$

e) $\dfrac{x}{\sqrt{x^2+25}}$

9. If $f(x) = \sin^2 e^x$, then $f'(x)$ is

a) $2\cos e^x$

b) $\cos^2 e^x$

c) $2\sin e^x \cos e^x$

d) $2xe^{x-1}\sin e^x \cos e^x$

e) $2e^x \sin e^x \cos e^x$

10. If $f(x) = \frac{1}{48}x^3 - 48x^{-\frac{1}{3}} + 16$, then $f'(8)$ is

a) $\frac{8}{3}$

b) 4

c) 5

d) 8

e) 260

11. Suppose $h = f/g$ and that f and g satisfy $f(2) = 36$, $f'(2) = -12$, $g(2) = 4$, and $g'(2) = 2$. Then $h'(2)$ has value

    a) 30

    b) 7.5

    c) 6

    d) -6

    e) -7.5

12. An equation for the line that is tangent to the graph of the function $f(x) = \sqrt[3]{x}$ at the point with coordinates $(-27, -3)$ is

    a) $y = \frac{1}{27}x + 2$

    b) $y = \frac{1}{27}x - 2$

    c) $y = -\frac{1}{27}x - 4$

    d) $y = -\frac{1}{27}x + 2$

    e) $y = \frac{1}{9}x$

13. Two variables u and v are related by the equation $u = 2v^3 - v$. The instantaneous rate of change in v with respect to u when $(u, v) = (-3, 1)$ is

    a) -51

    b) -3

    c) $\frac{1}{5}$

    d) 1

    e) 5

Questions 14 and 15 refer to a particle moving in a straight horizontal line whose position s at time t is given by the equation $s = 3t^3 - 4t^2 + t + 12$. The *positive* direction is to the *right*.

14. The velocity v and acceleration a at time $t = 2$ are

   a) $v = 21$, $a = 21$

   b) $v = 21$, $a = 28$

   c) $v = 22$, $a = 21$

   d) $v = 28$, $a = 18$

   e) $v = 28$, $a = 21$

15. As the time t increases from zero, the particle

   a) always moves to the left

   b) always moves to the right

   c) moves to the right, stops, and moves to the left from then on

   d) moves to the left, stops, and moves to the right from then on

   e) moves to the right, stops, moves to the left, stops again, and finally moves to the right from then on

16. An equation of the line that is tangent at the point with coordinates $(-1, 2)$ to the curve with equation $2x^3 - x^2y + 3y^3 = 20$ is

   a) $10x + 35y = 60$

   b) $-10x + 35y = 80$

   c) $10x + 35y = 80$

   d) $21x + y = -19$

   e) $46x - y = -48$

17. Let $f(x) = 3ax^4 + 8ax^3 + b$ where a and b are positive numbers. Then f has a global minimum when x is

a) $-\frac{4}{3}$

b) -2

c) $-\frac{2a}{b}$

d) 0

e) impossible to determine (Not enough information given.)

18. The best linear approximation to the value of $\sqrt{1+h}$ for h close to zero is

a) 1

b) $1 + \frac{1}{2}h$

c) $1 + h$

d) $1 + 2h$

e) $1 + h^2$

19. A patch of diseased plants in a field is roughly circular, with a *diameter* growing at a rate of about 5 feet per week. At approximately what rate is the area of the patch growing when its diameter is 60 feet? [Be sure to note the difference between *radius* and *diameter*.]

a) 236 sq.ft./week

b) 471 sq.ft./week

c) 942 sq.ft./week

d) 1414 sq.ft./week

e) 1885 sq.ft./week

20. If $f(x) = \dfrac{ax + b\tan x}{c\sin x}$ (a, b, and c fixed real numbers), then $\lim_{x\to 0} f(x)$ is

a) $\dfrac{a+b}{c}$

b) $\dfrac{a}{c}$

c) $\dfrac{a-b}{c}$

d) $\dfrac{a}{c}$

e) $+\infty$

21. If $f(x) = \dfrac{\ln(mx+b)}{\ln x}$ ($m > 0$, a fixed real number), then $\lim_{x\to\infty} f(x)$ is

a) 0

b) m

c) 1

d) $+\infty$

e) undefined (the limit does not exit)

22. The function $f(x) = \dfrac{ax^3}{x-b}$, $x > b$, arises in the study of migratory animals. (a and b are fixed, positive parameters.) It is concave up everywhere. Its minimum value is biologically important. The minimum value of f is

a) 0

b) $\frac{2}{3}b$

c) $\frac{3}{2}b$

d) $\dfrac{a}{1-b}$

e) ab

23. The two positive numbers whose product is 2001 and whose sum is as small as possible are

a) 1, 2001

b) $\sqrt{2001}$, $2001 - \sqrt{2001}$

c) $\frac{2001}{2}$, $\frac{2001}{2}$

d) $\frac{\sqrt{2001}}{2}$, 2

e) $\sqrt{2001}$, $\sqrt{2001}$

24. In the search for a root of a function f, the starting value for Newton's Method is $x_1 = 2$. If $f(2) = 0.3$ and $f'(2) = -1.2$, then one iteration of the algorithm produces

a) $x_2 = -2$

b) $x_2 = 1.75$

c) $x_2 = 2.25$

d) $x_2 = 2.3$

e) $x_2 = 6$

25. If $f'(x) = e^x + x^3 - x$ and $f(0) = 2$, then $f(2)$ is approximately

a) 0

b) 6.4

c) 7.4

d) 9.4

e) 10.4

END OF MULTIPLE CHOICE SECTION