

MATH 1241
COMMON FINAL EXAMINATION
FREE RESPONSE SECTION
FALL, 1996

This exam is divided into two parts. These pages contain Part II which consists of 6 free response questions.

Please show all of your work on the problem. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**

At the end of the examination you **MUST** hand in this test booklet and all scratch paper.

PROBLEM	1	2	3	4	5	6
GRADE						

FREE RESPONSE SCORE: _____

Name: _____ Student No: _____

Instructor: _____ Section No: _____

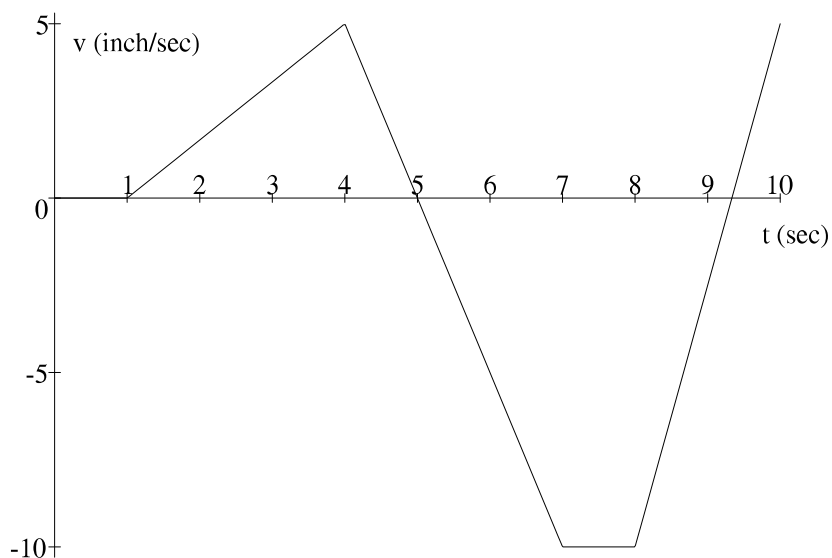
1. Let $f(x)$ be a continuous function on the interval $[-4, 4]$. The derivatives of $f(x)$ have the properties indicated in the following table.

x	$[-4, -2)$	-2	$(-2, 0)$	0	$(0, 1)$	1	$(1, 4]$
$f'(x)$	< 0	0	> 0	> 0	> 0	0	< 0
$f''(x)$	> 0	0	> 0	0	< 0	< 0	< 0

- (a) On what intervals is $f(x)$ increasing and on what intervals is it decreasing?
- (b) On what intervals is the graph of $f(x)$ concave up and on what intervals is it concave down?
- (c) Identify the x -coordinate for each local maximum and local minimum point of the graph of $f(x)$.
- (d) Give the x -coordinate of each inflection point of the graph of $f(x)$.
- (e) Draw a possible graph of $f(x)$ on $[-4, 4]$, assuming that $f(0) = 1$.

2. For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, let $f(x) = \frac{e^{x/2}}{1 + \sin(x)}$.

- (a) Sketch a plot of $f(x)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ **and** $-10 \leq y \leq 10$.
- (b) Find $f'(x)$.
- (c) Find an equation of the line tangent to the graph of f at the point where $x = 0$.
3. A particle is located at the point 0 on the x -axis at time $t = 0$. The particle moves along the x -axis in such a way that its velocity $v(t)$ at time t is given by the following graph.



- (a) Find the average velocity of the particle over the interval $0 < t < 6$.
- (b) At what positive time will the particle return to the point 0?
- (c) Find the maximal distance between the particle and its position at $t = 0$.
4. Let $f(x)$ be the function defined by $f(x) = e^{\frac{1}{x}-x}$ for $x \geq 0$.
- (a) Write an equation for the line tangent to the graph of $f(x)$ at the point where $x = 1$.
- (b) Using the equation you found in Part (a), approximate the value of $f(1.1)$.
5. Given the following table of data for the function f .

x	3.0	3.2	3.4	3.6
$f(x)$	6.3	5.6	5.1	4.7

- (a) Estimate $f'(3.4)$.
- (b) Suppose that $g(x) = \sqrt{x}$. Find the derivatives of $f(x) \cdot g(x)$ and $f(x)/g(x)$ at $x = 3.4$.

6. Let $f(x) = x \cdot e^{-bx}$, where b is a positive constant.

(a) Find b such that $f(x)$ has an absolute maximum value at the point $x = 2$.

(b) Find the intervals where the function $f(x)$ which you specified in Part (a) is concave up and where it is concave down.