MATH 1241
COMMON FINAL EXAMINATION
MULTIPLE CHOICE SECTION
SPRING, 1997

This exam is divided into two parts. These pages contain Part I which consists of 20 multiple choice questions. Part II consists of 6 free response questions You have three hours for the entire test. Part I should be completed in two hours. Part II should be completed in one hour.

This part of the exam consists of 20 multiple choice questions. They are printed on the front and back of each page. Be sure that you answer 20 different questions. A special answer sheet is provided so that your answers can be machine graded.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select among the choices the choice that **best approximates** the exact numerical value.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At the end of the examination you MUST hand in this test booklet, your answer sheet and all scratch paper.

1. The equation of the tangent line to the graph of the function $f(x) = \cos(x)$ at $x = \frac{\pi}{2}$ is:
   (a) $y = 0$
   (b) $y = x - 1$
   (c) $y = -x + \dfrac{\pi}{2}$
   (d) $y = -\sin(x)$
   (e) $y = x - \dfrac{\pi}{2}$

2. The derivative of the function $\dfrac{\sin(2x)}{1+x^2}$ is:
   (a) $\dfrac{\cos(2x)}{x}$
   (b) $\dfrac{\cos(2x)}{(1+x^2)^2}$
   (c) $\dfrac{\cos(2x)}{2x}$
   (d) $2\dfrac{\cos(2x)(1+x^2) - x\sin(2x)}{(1+x^2)^2}$
   (e) $\dfrac{\cos(2x)(1+x^2) - 2x\sin(2x)}{(1+x^2)^2}$

3. If $f'(x) = (x-1)(x-3)^2$, then the function f
   (a) is decreasing for $x > 1$.
   (b) is increasing for all x.
   (c) has a local minimum at $x = 3$.
   (d) has an inflection point at $x = 3$.
   (e) has none of the above.

4. The tangent line to the graph of a function f at the point where $x = 1$ has the equation $y = 2x - 1$. Then a good approximation for $f(1.1)$ is

(a) 1

(b) 2.2

(c) 1.2

(d) 2.1

(e) 2

5. $(e^{2\ln 2 - \ln 8 + 1})^2$ is equal to:

(a) e^2

(b) e^{-2}

(c) $2e^{-2}$

(d) $\frac{e^2}{4}$

(e) $\frac{e^{-2}}{4}$

6. As h approaches 0, $\frac{\sin(\frac{\pi}{6} + h) - \frac{1}{2}}{h}$ approaches:

(a) $\frac{1}{2}$

(b) $\frac{2\sqrt{2}}{3}$

(c) $\frac{\sqrt{3}}{2}$

(d) 0

(e) ∞

7. If $f(2) = 1,\ f'(2) = 2,\ f(3) = -1,\ f'(3) = 3,\ g(2) = 3$, and $g'(2) = 5$, then $\dfrac{d}{dx} f(g(\sqrt{x}))$ at the point $x = 4$ is:
   (a) $\dfrac{15}{4}$
   (b) $\dfrac{15}{2}$
   (c) 15
   (d) 10
   (e) 20

8. If $f(x) = 1 - \ln(x+2)$, then $e^{f(-1)-1}$ equals:
   (a) e
   (b) e^{-1}
   (c) $\ln 1$
   (d) 1
   (e) e^{-2}

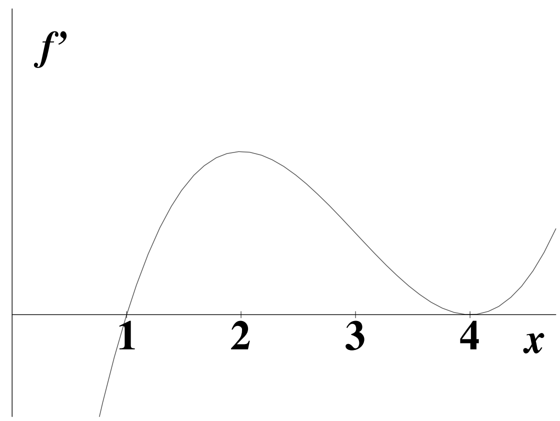
9. If the position of a particle on a straight line at time t is given by $x(t) = \ln(t/2)$ then the average velocity of the particle over the interval $[4, 8]$ is closest to:
   (a) 0.5
   (b) 0.25
   (c) 0.125
   (d) 0.06
   (e) 0.17

10. The global maximum value of the function xe^{-ax}, where $a > 0$, is:
    (a) ae^{-a^2}
    (b) $-ae^{-a^2}$
    (c) $(ae)^{-1}$
    (d) 0
    (e) e^{-1}

11. The equation of the straight line through the point $(-1, 2)$ which is parallel to the tangent line to $f(x) = \sqrt{4-x}$ at the point $x = 3$ is:
    (a) $y = -\frac{1}{2}x + 1$
    (b) $y = \frac{1}{2}x + \frac{5}{2}$
    (c) $y = -x + 3$
    (d) $y = -\frac{1}{2}x + \frac{3}{2}$
    (e) $y = 2x + 4$

12. If prices are increasing by 4% at the *end of each year*, then they will be doubled in:
    (a) 25 years
    (b) 17 years
    (c) 18 years
    (d) 16 years
    (e) 15 years

13. The graph of $f'(x)$, the derivative of $f(x)$, is given below



Which of the following is true?

I. f has a local maximum at $x = 1$.

II. f has a local minimum at $x = 1$.

III. f has a local maximum at $x = 2$.

IV. f has a local minimum at $x = 4$.

(a) III and IV only

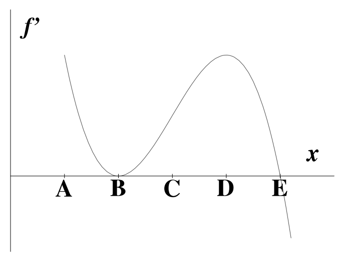
(b) II and III only

(c) II only

(d) III only

(e) I and IV only

14. The graph of $f'(x)$, the derivative of $f(x)$, is given below



Then f is concave up on which of the following intervals?

(a) (A, D)

(b) (B, D)

(c) (D, E)

(d) (A, E)

(e) (A, C)

15. If $f(x) = 2x + 1$ then the average value of f over the interval $[4, 8]$ is closest to:

(a) 6.5

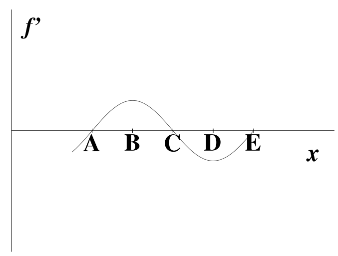
(b) 13

(c) 26

(d) 52

(e) 8

16. The graph of $f'(x)$, the derivative of $f(x)$, is given below.



Then the function $f(x)$ on the closed interval $[A, E]$ has its global maximum value at

(a) A

(b) B

(c) C

(d) D

(e) E

17. The right-hand Riemann sum approximation of the integral $\int_0^2 \sqrt{1+x^3}\,dx$ with 4 subdivisions is closest to:

(a) 2.783

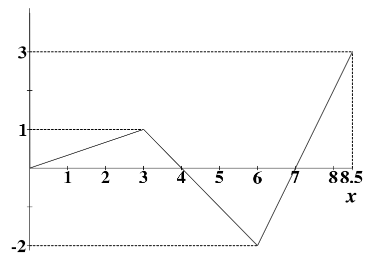
(b) 3.783

(c) 3.891

(d) 3.241

(e) 3.220

18. The function f is given by the graph below.



The integral $\int_0^7 f(x)\,dx$ is

(a) 10

(b) 5

(c) 1

(d) 0

(e) -1

19. The second derivative of $y = e^{\sin(2x)}$ is
    (a) $4e^{\sin(2x)}\left(\cos^2(2x) - \sin(2x)\right)$
    (b) $e^{\sin(2x)}$
    (c) $e^{2\cos(2x)}$
    (d) $2e^{\sin(2x)}\cos(2x)$
    (e) $e^{\sin(2x)}\left(\cos(2x) + \sin(2x)\right)$

20. The largest value of the slope of the graph of the function $y = e^{-x^2}$ is:
    (a) $1/\sqrt{e}$
    (b) 1
    (c) $2e^{-1}$
    (d) $\sqrt{2/e}$
    (e) $\sqrt{e/2}$

END OF MULTIPLE CHOICE SECTION