MATH 1242
Common Final Examination
Free Response Section
Fall, 1998

This exam is divided into two parts. These pages contain Part II which consists of 6 free response questions.

Please show all of your work on the problem. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**
- The table of integrals from the text appears at the end of this examination booklet. You may find these useful.

At the end of the examination you MUST hand in this test booklet and all scratch paper.

PROBLEM	1	2	3	4	5	6
GRADE						

FREE RESPONSE SCORE:_____

Name: _____ Student No: _____

Instructor: _____ Section No: _____

1. Use Euler's method with step size 0.25 to approximate the value $y(1)$ for the solution of the initial-value problem. Show your work.

$$y' = x^2 - y, \qquad y(0) = 1.$$

x-value	y-value	
0	1	

2. Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the temperature difference between the object and its surroundings. A glass of orange juice is taken from a 40°F refrigerator and placed in a 65°F room. The orange juice warms to a temperature of 50°F in one hour.

   (a) Write a differential equation with an initial condition, that describes the temperature of the orange juice at time $t > 0$.

   (b) Solve this differential equation.

   (c) How long will it take for the orange juice to warm to 60°F?

3. Find the following integrals

(a) $\int_{-1}^{1} x^2(1+x^3)^3 dx$

(b) $\int \frac{12}{4+x^2} dx$

(c) $\int x^3\sqrt{x^2-4} dx$

(d) $\int \frac{2x}{x^2-4} dx$

4. The function F is defined on the interval $[0, 2\pi]$ by

$$F(x) = \int_0^x t\cos(2t)dt.$$

(a) Find the rate of change of the function F as a function of x.

(b) Find the maximum value of F on the interval $[0, 2\pi]$.

(c) Set up an integral for the arclength of $F(x)$ on the interval $[0, 3]$. Use your calculator to estimate the value to 2 decimal places.

5. Find the volume of the solid obtained by revolving the region bounded by the curve $y = 2x^2 - x^3$ and the x-axis

   (a) about the x-axis.

   (b) about the y-axis.

6. Recall that the Maclaurin series of a function is the Taylor series expansion of the function about zero.

   (a) Find the Maclaurin series for the function $f(x) = e^x$.

   (b) Find the Maclaurin series for the function $f(x) = e^{-x}$.

   (c) The hyperbolic sine function is defined by

   $$\sinh x = \frac{e^x - e^{-x}}{2}.$$

   Find its Maclaurin series.

   (d) Find the Maclaurin series for $\dfrac{d}{dx} \sinh x$.