

MATH 1242
COMMON FINAL EXAMINATION
FREE RESPONSE SECTION
FALL, 1999

This exam is divided into two parts. These pages contain Part II which consists of 6 free response questions.

Please show all of your work on the problem. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**
- The table of integrals from the text appears at the end of this examination booklet. You may find these useful.

At the end of the examination you **MUST** hand in this test booklet and all scratch paper.

PROBLEM	1	2	3	4	5	6
GRADE						

FREE RESPONSE SCORE: _____

Name: _____ Student No: _____

Instructor: _____ Section No: _____

1. Let $F(x) = \int_0^{2x} f(t) dt$

(a) Compute $D_x F(x) = \frac{d}{dx} F(x)$.

(b) Compute $F(x)$ in the special case when $f(t) = \cos(t) \sin(t)$.

(c) Compute $D_x F(x)$ where $F(x)$ is your answer to part (b).

(d) Use part (a) to determine $D_x F(x)$ when $f(t) = \cos(t) \sin(t)$.

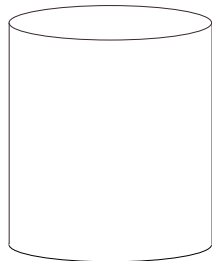
2. Given the differential equation

$$\frac{dy}{dx} = y^2(1 + x)$$

- (a) Find all solutions to this equation.
- (b) Find the solution that satisfies the initial condition $y(1) = 2$

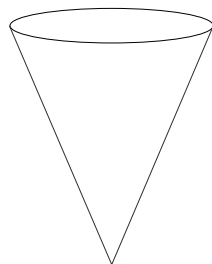
3. Let A be the region between the curves $y = x$ and $y = x^2$.
- (a) Find the definite integral that represents the volume of the solid generated by rotating A about the x -axis
- (b) Evaluate this integral
- (c) Find the definite integral that represents the volume of the solid generated by rotating A about the line $x = -1$.
- (d) Evaluate this integral.

4. A water tank has the form of a cylinder with radius 8 feet and height 20 feet. The tank is full of water. Note that one cubic foot of water weighs 62.4 pounds.
- (a) If the cylinder is sitting upright, find the definite integral that represents how much work it takes to pump all the water out of the top of the tank.



- (b) Evaluate the integral you found above.

- (c) The engineers decided that a cylindrical tank would weigh too much. They think that a tank in the shape of a cone with a top radius of 8 feet and a height of 20 feet would work better. Find the definite integral that represents how much work it takes to pump all the water out of the top of the tank.



- (d) Evaluate the integral you found above.

5. Consider the differential equation $y' = 2 - y$ with initial value $y(0) = 1$.

(a) Use Euler's method with three steps of size $1/3$ to estimate $y(1)$.

x	y
0	
$1/3$	
$2/3$	
1	

(b) Show that the exact solution to this initial value problem is $y(x) = 2 - e^{-x}$.

(c) Compare the exact value of $y(1)$ with the estimate you computed in part (a). Was your estimate an overestimate or an underestimate?

6. Let A be the region between the curves $y = x$ and $y = x^2$.

(a) Draw a sketch of the region A .

(b) Using 5 rectangles of equal width, compute the Left-hand Riemann sum that approximates the area of A .

(c) Find the area of A *exactly* by integration.

(d) Will doubling the number of rectangles decrease your error by a factor of 2? Why or why not?