

MATH 1242  
COMMON FINAL EXAMINATION  
FREE RESPONSE SECTION  
SPRING, 1997

This exam is divided into two parts. These pages contain Part II which consists of 6 free response questions.

Please show all of your work on the problem. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**
- The table of integrals from the text appears at the end of this examination booklet. You may find these useful.

At the end of the examination you **MUST** hand in this test booklet and all scratch paper.

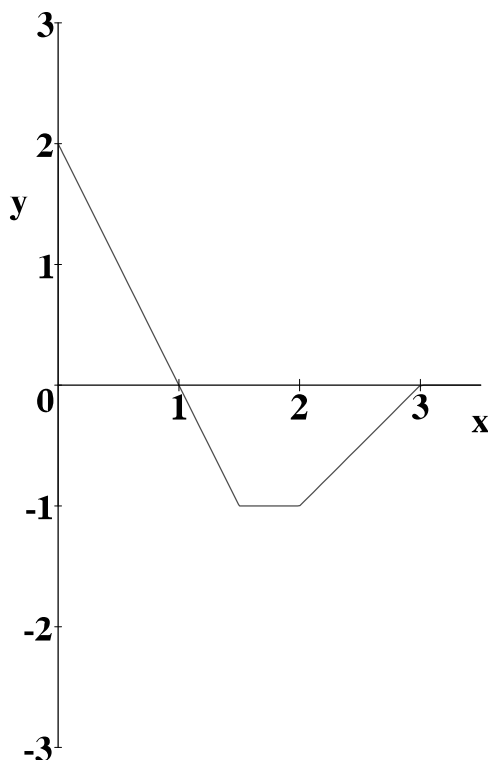
PROBLEM	1	2	3	4	5	6
GRADE						

FREE RESPONSE SCORE: \_\_\_\_\_

Name: \_\_\_\_\_ Student No: \_\_\_\_\_

Instructor: \_\_\_\_\_ Section No: \_\_\_\_\_

1. The graph of the derivative  $f'$  of some function  $f$  is shown in the following figure. Sketch the graph of the function  $f$  and explain your reasoning. (You may assume  $f(0) = 0$ .)



2. The three statements below refer to errors involved when approximating  $\int_a^b f(x) dx$ . In each case determine whether the given statement is true or false and explain your reasoning.
1. The error is generally less with the midpoint rule than with Simpson's rule.
  2. The error in the left rule depends on  $f'$  and  $n$ .
  3. The trapezoid rule approximation to  $\int_0^1 x^2 dx$  is always smaller than the exact value of the integral.

3.

(a) Find the indefinite integral  $\int \frac{dx}{x^2 - 3x}$ .

(b) Sketch the graph of the integrand  $f(x) = \frac{1}{x^2 - 3x}$ .

(c) For which values of  $x$  is  $f(x)$  undefined?

(d) Find  $\int_4^\infty \frac{dx}{x^2 - 3x}$  if it exists.

(e) Find  $\int_1^3 \frac{dx}{x^2 - 3x}$  if it exists.

4.

(a) Sketch the graph of  $y = -2\cos(x) + 2$  from  $x = -\pi$  to  $x = \pi$ .

(b) Shade in the region bounded by this curve, the  $x$ -axis,  $x = 0$  and  $x = 2$ .

(c) Set up the integral that represents the volume of the solid obtained by rotating the region about the  $x$ -axis.

(d) Find this volume. Explain how you arrived at your answer.

5. You are given the differential equation  $\frac{dy}{dx} = 3x^2$ ,  $y(1) = 3$ .

(a) Use Euler's method to approximate the value of  $y$  when  $x = 2$  on the solution curve that passes through  $(1, 3)$ . Use  $\Delta x = 0.25$  and round your answer to three decimal places.

(b) Solve the differential equation to find the exact value of  $y$  when  $x = 2$ .

(c) Was your approximation too big or too small? Explain why this happened.

6. An anticoagulant drug is prescribed for a patient. After stopping administration of the drug, the quantity remaining in the patient's body decreases at a rate proportional to the quantity remaining. The half-life of the drug is 24 hours.

(a) Sketch a graph of the quantity,  $Q$ , of the drug in the patient's body as a function of time,  $t$ , since administering 100 mg of the drug. Label the points on your graph where  $t = 0$ ,  $t = 24$ , and  $t = 48$ .

- (b) How many hours does it take for the drug level to be reduced to 10% of the original level?