MATH 1242
Common Final Examination
Free Response Section
Spring 1999

This exam is divided into two parts. These pages contain Part II which consists of 6 free response questions.

Please show all of your work on the problem. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**
- The table of integrals from the text appears at the end of this examination booklet. You may find these useful.
- **WARNING: The use of a TI-89 or TI-92 calculator on this exam is a violation of the Student Code.**

At the end of the examination you MUST hand in this test booklet and all scratch paper.

PROBLEM	1	2	3	4	5	6
GRADE						

FREE RESPONSE SCORE:_____

Name: _____ Student No: _____

Instructor: _____ Section No: _____

1. Compute the following integrals

a) $\int \frac{\ln x}{x^2}\,dx$ (HINT: Use integration by parts)

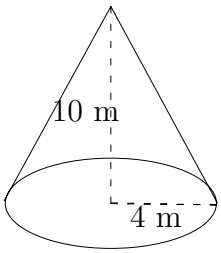
b) $\int \sec^2(2x) \cdot \tan(2x)\,dx$

c) $\int x\sqrt{x+3}\,dx$ (HINT: Use the substitution $u = x + 3$)

2.

a) Set up an integral for the area of the region R bounded by the line $y = x/2$ and the parabola $y^2 = 8 - x$. (HINT: Sketch the region R and integrate with respect to y.)

b) Consider the solid generated by revolving around the y-axis the region under $y = 3x^2$ from $x = 0$ to $x = 3$. Set up an integral for the volume of this solid.



3. A tank has the shape of an inverted circular cone with height 10 m and base radius 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m^3).

a) Set up an integral for the work required to empty the tank by pumping all of the water out of the top of the tank.

b) Evaluate the integral.

4. a) Solve the following initial value problem.

$$\frac{dy}{dx} = \frac{1+x}{xy}, \quad x > 0, \quad y(1) = -4$$

b) Suppose $\frac{dy}{dx} = x + y^2$ and $y(0) = 1$. Use Euler's method woth $h = 0.2$ to approximate $y(0.2)$, $y(0.4)$, and $y(0.6)$. Show the details of your calculation not just the numerical values.

$y(0.2) =$

$y(0.4) =$

$y(0.6) =$

5.

a) Find the power series for $\ln(1+x)$ about 0.

b) Determine the interval of convergence for Part (a).

c) Use Part (a) to find the power series about 0 for $f(x) = \int_0^x \ln(1+t)\,dt$.

6. A bacteria culture starts with 300 bacteria and grows at a rate proportional to its size.

a) After 3 hours there are 800 bacteria. Find $y(t)$ the number of bacteria in the culture at time t.

b) How much time in hours will it take for the population to reach 2000 bacteria.