ELEMENTS OF STATISTICS I
COMMON FINAL
SPRING 1993

PLEASE PRINT THE FOLLOWING INFORMATION:

NAME: _____ INSTRUCTOR: _____

STUDENT ID: _____ SECTION/TIME: _____

THIS EXAM HAS TWO PARTS.

PART I.Consists of 25 multiple choice questions (4 points each). Read all questions carefully. You may do calculations on the test paper. Mark the number of the opscan sheet corresponding to the test question number with a Number 2 pencil or a mechanical pencil with an HB lead. Mark only one answer; otherwise the answer will be counted as incorrect. In case there is more than one answer, mark the best answer. Please make sure that your name appears on the opscan sheet in the spaces provided.

PART II.This part consists of 3 questions (20 points each). You must show all work for each question in the space provided to receive full credit for that question. If you write your explanations in another part of the test, please indicate accordingly.

FOR DEPARTMENTAL USE ONLY:

Part II

Questions	1	2	3	4
Score				

Part I	Part II	TOTAL

STAT 122X
Common Final Examination
Spring 1993

Part I. Multiple choice. Each correct response is worth 4 points.
The following is used for questions 1, 2, 3, and 4.
Here is a sample from some population: 3, 0, 6, 0, 1, 0, 2, 10

1. The mean of these data is

   a) 4.5
   b) 3.25
   c) 2.75
   d) 9
   e) none of these

2. The median is

   a) 1.5
   b) 5.5
   c) 5
   d) 0
   e) none of these

3. The sample standard deviation is

   a) 18.75
   b) 3.58
   c) 3.2
   d) 2
   e) none of these

4. The range is

   a) 0
   b) 7
   c) 12.5
   d) 10
   e) none of these

5. Let X be a normal random variable with mean 2 and variance $\frac{1}{4}$. $P(X \geq 2.45)$ is best approximated by

   a) .3159
   b) .1841
   c) .4641
   d) .0359
   e) none of these

The following is used for Questions 6-8.

The table below gives a breakdown by age and annual income of people responding to a certain poll.

INCOME (in thousands) AGE (in years)	$< \$30$	$\$30 - \$50$	$> \$50$
< 25 years	7	10	10
$25 - 50$ years	10	27	16
> 50 years	4	13	3

Suppose one person who responded is selected at random.

6. What is the probability that the person selected is at least 25 years old and earns $30,000 or more?

   a) .2
   b) .5
   c) .27
   d) .53
   e) .59

7. What is the probability that the person selected earns $50,000 or less?

   a) .29
   b) .71
   c) .17
   d) .37
   e) .5

8. Given that the person selected earns less than $30,000, what is the probability that he is less than 25 years old?

   a) .21
   b) .27
   c) .8
   d) .33
   e) .07

The following is used for Questions 9 and 10.

A researcher calls a random sample of 10 households in a community on the night that 80% of the households have someone at home.

9. What is the probability that the researcher finds someone at home in at least 7 households?

   a) .560
   b) .624
   c) .322
   d) .121
   e) .878

10. What would be an expected number of successful calls?

a) 4
b) 8
c) .2
d) 10
e) 12

The following is used for Questions 11-13.

Let X be a discrete random variable with the following probability function

x	-1	1	2
$p(x)$	.2	.3	.5

11. What is $P(0 < X < 3)$?

a) .3
b) 1
c) .8
d) .5
e) .7

12. The mean μ of X is

a) 4
b) 1.5
c) -1
d) 2.7
e) 1.1

13. The variance σ^2 of X is

a) 2.50
b) 1.21
c) 1.29
d) 2
e) 1.73

The following is used for Questions 14-16.

A medical measurement is subject to error. The errors in the scale's readings are normally distributed with mean equal to 1 and standard deviation equal to 7.

14. What is the probability that a single measurement error will be greater than zero?

a) .0764
b) .9236
c) 1
d) .5
e) .5557

15. Suppose that 25 measurements are selected at random. What is the standard deviation of the sample mean of the 25 errors?

    a) 7
    b) .28
    c) 1.4
    d) .01
    e) 5

16. What is the probability that the sample mean of 25 measurement errors is greater than zero? (round up to four decimal places)

    a) .5160
    b) .4236
    c) .7612
    d) .5
    e) .95

17. A manufacturer interested in estimating the proportion of defective items it produces tested a sample of 400 items and found 25 to be defective. Find a 95% confidence interval for p, the true fraction of defective items.

    a) $(3.42, 3.56)$
    b) $(.305, .367)$
    c) $(.039, .086)$
    d) .95
    e) $(.043, .082)$

18. Set up a 90% confidence interval for the mean time it takes to get from UNCC to Coliseum if in a random sample of 60 trips the sample mean was 40 minutes and the sample standard deviation was 8 minutes.

    a) $(26.84, 53.16)$
    b) $(38.3, 41.7)$
    c) $(52, 68)$
    d) $(38.68, 41.32)$
    e) $(42.5, 47.8)$

    The following is used for Questions 19-21.

    Over a long period of time in which the posted speed limit was 65 mph, the average speed along a certain stretch of highway was found to be 63.0 mph. After the speed limit dropped to 55 mph, the average speed of cars in a sample of size 100 was found to be 61.9 mph, and the standard deviation, $s = 4.6$ mph.

19. State the null and alternative hypothesis in a test to determine if the data provide sufficient evidence of a genuine reduction in mean speed.

    a) H_0: $\mu = 65$

b) H_0: $\mu < 61.9$

c) H_0: $\mu = 65$

d) H_a: $\mu = 55$, H_a: $\mu \geq 61.9$, H_a: $\mu < 65$

e) H_0: $\mu = 63$
H_a: $\mu < 63$

20. The value of the test statistic in this example is

a) $-.08$

b) .04

c) -3.48

d) -2.39

e) none of these

21. The observed significance level (p-value) of the test is

a) .0319

b) .0084

c) .484

d) .3632

e) none of these

22. How large a sample size should one use to estimate p from a binomial population to within 0.1 at a 95% confidence level?

a) 69

b) 463

c) 1024

d) 97

e) 216

23. Independent random samples of 81 observations are chosen from each of two normal populations with the following means and standard deviations

Population 1	Population 2
hline $\mu_1 = 15$	$\mu_2 = 15$
$\sigma_1 = 4$	$\sigma_2 = 6$

The standard deviation of $\bar{x}_1 - \bar{x}_2$ is

a) 10

b) 7.20

c) 1.49

d) 2.73

e) .80

The following is used for Questions 24 and 25.

A coin-operated coffee machine dispenses a preset amount of coffee. The vending company wants to know if the average amount dispensed is $\mu = 6$ (H_0, no adjustment needed) or is $\mu \neq 6$ (H_a, requires adjustment).

24. Which of the following is an appropriate rejection region for conducting the above test at significance level .05 if sample size is equal to 50?

    a) $Z > .17$ or $Z < -.17$

    b) $Z > 1.96$

    c) $Z < 1.645$

    d) $Z > 1.96$ or $Z < -1.96$

    e) $Z > 1.282$ or $Z < -1.282$

25. Suppose the observed value of the test statistic is 1.53. What is the smallest level for which there is sufficient evidence to reject H_0 (*i.e.* the p-value for this test)?

    a) 1.96

    b) .126

    c) -1.96

    d) .063

    e) .05

PART II

1. To determine which students should receive scholarships, a university admissions officer decided to study the relationship between a student's score on the SAT verbal test (taken in the final year of high school) and the student's college GPA at the end of the sophomore year. Ten student records were examined with the following results. The reported exam scores are the actual scores divided by 100.

Student	SAT	GPA
1	4.8	2.4
2	6.6	3.5
3	5.9	3.0
4	7.4	3.8
5	3.8	2.7
6	5.2	2.4
7	6.6	3.0
8	5.0	2.8
9	7.2	3.4
10	6.0	3.2

Let x be SAT score and y be GPA. Then $\sum x^2 = 354.05$, $\sum y^2 = 93.14$, $\sum xy = 180.66$, $\sum x = 58.5$, and $\sum y = 30.2$.

a) Find the least squares regression line. (8 pts)

b) Find the predicted GPA for the first student in the sample. (2 pts)

c) Find the correlation coefficient. (3 pts)

d) Find a 95% confidence interval for the mean GPA of students who score 800 ($x = 8$) on the SAT, based on these data. (7 pts)

2. It was claimed that the mean grade point average in University A was not the same as the mean in University B. Independent random samples of 18 students attending University A and 14 students attending University B gave the following statistics for GPA. Assume normal distribution for the distribution of GPA in both universities and that variances are equal.

UNIVERSITY A	UNIVERSITY B
$\bar{x}_1 = 3.64$	$\bar{x}_2 = 3.78$
$s_1 = .42$	$s_2 = .38$

a) State the null and alternative hypotheses. (5 pts)

b) Carry out the test at $\alpha = .05$. State your conclusion. (8 pts)

c) Find a 95% confidence interval for $\mu_1 - \mu_2$. (7 pts)

3. A pollster surveys 150 voters and finds that 56 of them favor candidate A.

a) What point estimate do the data provide for the proportion p of all voters who favor A? (4 pts)

b) Construct 95% confidence interval for the proportion p. (8 pts)

c) What is the maximum error of his estimate? (4 pts)

d) How the pollster must change the sample size to reduce the maximum error of estimate by 50%. (4 pts)