Advanced Functions and Modeling

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Updated 05/21/03

Advanced Functions and Modeling

Advanced Functions and Modeling provides students an in-depth study of modeling and applying functions. Home, work, recreation, consumer issues, public policy, and scientific investigations are just a few of the areas from which applications should originate. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- ***Describe graphically, algebraically and verbally phenomena as functions; identify independent and dependent quantities, domain, and range, and input/output.***
- ***Translate among graphic, algebraic, numeric, and verbal representations of relations.***
- ***Define and use linear, quadratic, cubic, and exponential to model and solve problems.***
- ***Use systems of two or more equations or inequalities to solve problems.***
- ***Use the trigonometric ratios to model and solve problems.***
- ***Use logic and deductive reasoning to draw conclusions and solve problems.***

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GOAL 1: The learner will analyze data and apply probability concepts to solve problems.

1.01 Create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, and logarithmic functions of bivariate data to solve problems.
a) Interpret the constants, coefficients, and bases in the context of the data.
b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions and make predictions.

1.02 Summarize and analyze univariate data to solve problems.
a) Apply and compare methods of data collection.
b) Apply statistical principles and methods in sample surveys.
c) Determine measures of central tendency and spread.
d) Recognize, define, and use the normal distribution curve.
e) Interpret graphical displays of univariate data.
f) Compare distributions of univariate data.

1.03 Use theoretical and experimental probability to model and solve problems.
a) Use addition and multiplication principles.
b) Calculate and apply permutations and combinations.
c) Create and use simulations for probability models.
d) Find expected values and determine fairness.
e) Identify and use discrete random variables to solve problems.
f) Apply the Binomial Theorem.

GOAL 2: The learner will use functions to solve problems.

2.01 Use logarithmic (common, natural) functions to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

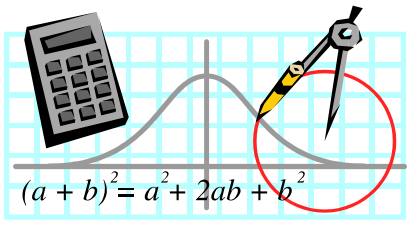
2.02 Use piecewise-defined functions to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

2.03 Use power functions to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

2.04 Use trigonometric (sine, cosine) functions to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
c) Develop and use the law of sines and the law of cosines.

2.05 Use recursively-defined functions to model and solve problems.
a) Find the sum of a finite sequence.
b) Find the sum of an infinite sequence.
c) Determine if a given series converges or diverges.
d) Translate between recursive and explicit representations.

July 2003 update from the UNC Office of the President and the UNC Minimum Course Requirement Mathematics Steering Committee:



Frequently Asked Questions

regarding the new fourth course mathematics requirement for admission to UNC institutions beginning fall 2006:

Question 1: I've just seen the brochure on the increase of minimum mathematics course requirements for UNC admission. You've listed seven courses in the brochure that meet the 4th course requirement (from AP Calculus to Advanced Functions and Modeling). Are there other courses that would meet this requirement, such as Trigonometry, Probability and Statistics, and Algebra III?

No, only these seven courses will meet the new UNC 4th course mathematics admission requirement: AP Calculus, AP Statistics, Pre-Calculus, Discrete Mathematics, IB Mathematics Level II, Integrated Mathematics IV, and Advanced Functions and Modeling. To view the goals and objectives of the standard course of study for six of these courses, visit the Web site:

http://www.learnnc.org/DPI/instserv.nsf/Category7.

Click on "Mathematics Curriculum," then on "2003 Mathematics Standard Course of Study," and then on "High School." See pp. 56, 57 of the document. IB Mathematics, Level 2 is not available on this site.

If school systems choose to offer any advanced mathematics courses other than these seven, such as Trigonometry, non-AP Probability, non-AP Statistics, and Algebra III, it is of the utmost importance that high school administrators, counselors, students, and parents be aware that these other courses do not meet the new UNC math admission requirement. Please note that the 2003-2004 sophomore class is the first group subject to the new math requirement that takes effect in fall 2006.

Question 2: Suppose a student takes Algebra I in 8th grade, and then Geometry, Algebra II, and Pre-Calculus in high school and then no math during their senior year. They have 4 years of "high school" mathematics, but only 3 units of high school credit. Will this count for admission to a UNC school?

Yes, the UNC requirement will be met. In addition, the UNC System highly recommends that students take a challenging math course during their senior year of high school.

However, be aware that the NC Department of Public Instruction (NCDPI) high school graduation requirements for the college/university prep course of study are not met unless a student completes 4 units of college/university prep math in grades 9-12. For more information, go to http://www.ncpublicschools.org/student_promotion/gradreq.html.

Question 3: Suppose a student takes Algebra I, Algebra II, Pre-Calculus, and Discrete Math in grades 9-12, but for some reason doesn't take Geometry. Will this sequence of courses satisfy the UNC requirements?

Yes, credit for Algebra I and Algebra II, plus any two of the seven approved advanced mathematics courses will satisfy minimum course requirements for admission to the UNC System.

However, current NCDPI graduation requirements for the college/university prep course of study do require Geometry, along with Algebra I and Algebra II, for students who do not choose the Integrated Mathematics series. Go to http://www.ncpublicschools.org/student_promotion/gradreq.html for more information.

Question 4: Can students receive credits toward high school graduation for both Alg III/Trig AND Advanced Functions and Modeling?

Credit toward high school graduation for both Alg III/Trig and Advanced Functions and Modeling might be received depending upon the course of study followed by the student. Please visit the Web site mentioned in question 3 for more information. **Alg III/Trig**, by itself, would **NOT** meet the UNC Admissions requirement because it is not on the list of seven acceptable advanced math courses.

Question 5: Will the math faculty of non-public high schools have the same opportunities for staff development offered by the NC Mathematics and Science Network (NC MSEN) centers for the new Advanced Functions and Modeling course? Will there be a tuition charge?

Information about NC MSEN staff development will be disseminated to all public and non-public high school math departments during the 2003-2004 school year. The cost involved will depend upon the source of funding obtained

for this training. Information about the cost will be available from each of the sites offering this staff development. Visit the NC MSEN Web site: http://www.unc.edu/depts/msen for more information.

Question 6: Our math faculty also needs staff development for the other fourth year courses, especially Discrete Math. Will this be offered at NC MSEN centers as well? If so, when? Will there be a fee involved? Will continuing education credit be available?

*There is a significant overlap in the course objectives taught in Pre-Calculus, Discrete Math, Integrated Mathematics 4, and Advanced Functions and Modeling. The staff development planned for the **Advanced Functions and Modeling** course during the summers of 2004 and 2005 by NC MSEN centers and the NC School of Science and Math would be beneficial to teachers of any of these four courses. If grant monies are received, fees will be covered for this staff development. CEUs will also be awarded.*

Also, feel free to contact the NC MSEN center in your region for staff development offerings specifically designed for Discrete Math, Pre-Calculus, Integrated Mathematics, AP Calculus, AP Stats, etc. Visit the NC MSEN Web site: http://www.unc.edu/depts/msen *for more information.*

Question 7: My math faculty is concerned about the fact that a course in Trigonometry has not been included in the list of seven acceptable courses. We do not feel there is enough trig covered, especially at the standard level, in the courses included in the acceptable course list. How can we better prepare and indicate in course title that we have covered a trig curriculum?

Trigonometry topics are included in the standard course of study for Advanced Functions and Modeling, Discrete Math, Integrated Mathematics, and Pre-Calculus. However, the standard course of study identifies only the minimum course requirements. Teachers may, at their discretion, include more in-depth treatment of trig topics.

The UNC System believes that the students in Advanced Functions and Modeling are bettered served by the variety of topics included in the course's newly created curriculum.

Question 8: What are the SIMS (Student Information Management System) numbers of the seven acceptable 4th math courses for UNC Admission?

	Course Title	SIMS #
1.	*AP Calculus**	*AB: 2076 / BC: 2077*
2.	*AP Statistics**	*2065 (indicate AP in SIMS)*
3.	*Pre-Calculus*	*2070*
4.	*Discrete Mathematics*	*2050 (indicate standard or honors in SIMS)*
5.	*IB Mathematics Level II*	*2078*
6.	*Integrated Mathematics IV*	*2054 (indicate standard or honors in SIMS)*
7.	*Advanced Functions and Modeling*	*2025*

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Be aware that local course numbers may differ from the state course codes listed above. The state course codes are used for electronic transcripts.

Question 9: When does our school system need to revise and rename a standard-level fourth course as *Advanced Functions and Modeling*?

*As soon as possible!! All high schools should offer a standard-level **Advanced Functions and Modeling** course in their mathematics curriculum by 2005-2006. If students pre-register for classes the previous spring, the **Advanced Functions and Modeling** course description and information should be included in the high school's curriculum handbook by spring 2005.*

It is the recommendation of the UNC Minimum Course Requirement Mathematics Steering Committee that advanced math courses offered to next year's juniors be among the approved seven courses, especially since this is the first group that must satisfy the requirement.

Question 10: Although the new UNC math requirement does not go into effect until fall 2006, math textbooks will be chosen by high schools this year (2003-04). Is there a list of acceptable textbooks for the new *Advanced Functions and Modeling* course?

*The new list of state adopted textbooks is to be approved by the State Board of Education on Oct. 1, 2003. When searching for a text for **Advanced Functions and Modeling**, high school math departments should be aware that texts appropriate for Pre-Calculus, Integrated Math 4, or Discrete Math may also serve their curriculum needs for Advanced Functions and Modeling.*

Also, the UNC Minimum Course Requirement Mathematics Steering Committee will be reviewing additional texts and support materials. The committee plans to make this list of recommended materials available to all high school math departments during the fall of 2003.

Question 11: In our school system, AP Calculus or AP Statistics credit is given only if a student takes the AP exam. If not, the student receives only honors credit. Honors Calculus and Honors Statistics are not listed as acceptable 4th math courses. What happens now to students who only receive honors credit?

Honors Calculus and Honors Statistics do not meet the new UNC admission requirement.

Question 12: We presently have two standard-level fourth year courses: Senior Review Class (for our slowest students - those who made a "D" in Algebra II) and Algebra III (for those who did well in Algebra II but are not quite ready for Honors Pre-Calculus. The Senior Review Class was renamed *Advanced Functions and Modeling*. What do we rename our Algebra III?? We want these students to meet the admission requirement too.

*The UNC Minimum Course Requirement Mathematics Steering Committee, in collaboration with NCDPI, designed **Advanced Functions and Modeling** as a standard-level course for college bound seniors for whom other advanced mathematics courses may not be appropriate or of interest. A standard-level Discrete Math course is a second option for the kinds of students described in this question. Please note that when course titles are changed to conform to the standard course of study, it is essential that course content also be changed accordingly.*

Question 13: Can students take Discrete Math and then Advanced Functions and Modeling?

Yes, and both courses would meet the new UNC admissions requirement. Honors or standard-level Discrete Math, and standard-level Advanced Functions and Modeling are acceptable courses.

Please keep in mind that Advanced Functions and Modeling was not designed to be taken in combination with either Discrete Math or Pre-Calculus. There is a great deal of duplication in the course objectives of these three courses. If a student has completed Advanced Functions and Modeling and needs or wants another advanced math course, the most appropriate of the seven approved courses is probably AP Statistics.

Question 14: Who can we call at the Department of Public Instruction or the University of North Carolina Office of the President if we have questions about the new math admissions requirement?

DPI: Bill Scott, Secondary Math Consultant, 919-807-3842
UNC-OP: Bobby Kanoy, Associate Vice Pres. for Academic Affairs, 919-962-1000

Exponential Model:

Credit Card Debt: A credit card for students advertises an annual percentage rate of 1.9%, with finance charges calculated using simple interest on the monthly balance. The minimum payment on the credit card is 3% of the outstanding balance or $15, whichever is greater – unless a balance of less than $15 remains, in which case your final payment equals this final balance plus the interest due on it. Suppose your credit card balance is $1,000 on Sept 1, 2003, and that you make only the minimum payment on the first of each month thereafter. If you make no further purchases on the card, when will you pay off the account? How much interest will you have paid over that time period?

Use a spreadsheet program to set up a payment schedule:

	A	B	C	D
1	Date	Balance	Interest Due	Payment
2	9/1/2003	$1,000.00	$1.58	$30.00
3	10/1/2003	$971.58	$1.54	$29.15
4	11/1/2003	$943.97	$1.49	$28.32
	⋮	⋮	⋮	⋮
	2/1/2008	$79.34	$0.13	$15.00
	3/1/2008	$64.46	$0.10	$15.00
	4/1/2008	$49.57	$0.08	$15.00
	5/1/2008	$34.64	$0.05	$15.00
	6/1/2008	$19.70	$0.03	$15.00
	7/1/2008	$4.73	$0.01	$4.74

This gives the student an opportunity to learn that in Excel you can generate the data by:

DATE: A3 = DATE(YEAR(A2),MONTH(A2)+1,DAY(A2))
Interest C3 = (0.019/12)*B3
Payment D3 =IF(B3<=15,B3+D3,MAX(0.03*B3,15))
Balance B4 = B3-C3+D3

Buying a Used Car:

When you buy a car or a house, your monthly payment is calculated by a method called amortization. Amortization is the process of paying off debt by making a given number of equal payments at specified intervals (usually monthly). These payments include the compound interest. With each payment, the amount of interest declines as the unpaid balance on the loan declines, while the amount paid toward principal increases. If equal payment are made monthly, then the payment amount is calculated according to the following formula:

$$\text{payment} = (\text{loan amount}) \times \frac{\text{interest rate}}{12} \times \frac{1}{1-\left(1+\frac{\text{interest rate}}{12}\right)^{-12t}}$$

where *t* is the number of years to repay the loan.

Let *P* represent the amount borrowed (the principal), and *m* represent the *monthly* interest rate (that is the APR/12). Then your monthly payment is given by:

$$\text{payment} = \frac{P \cdot m}{1-(1+m)^{-12t}}$$

Your parents agree to lend you $5,000 at 1.2%, amortized over 3 years. Your monthly interest rate is 0.012/12 = 0.001, and your monthly payment is

$$\text{payment} = \frac{(5000)(0.001)}{1-(1+0.001)^{-(12)(3)}} = \frac{5}{1-1.001^{-36}} = \$141.47$$

How do you figure out how much of each payment goes to interest and how much to principal? Each month you must calculate the interest on the current loan balance. If the monthly interest rate is 0.001 and the initial balance is $5000, then the first month's interest is $5. So of the first payment, only $136.47 is applied toward the principal, leaving a new balance of $4,863.53.

Using the monthly interest rate of 0.001 on this new balance gives a second month's interest of $4.86. So, $136.61 is applied toward principal, leaving a new balance of $4,726.92.

Continuing in this manner you can construct an *amortization schedule* for the loan. An amortization schedule gives the amount of each payment that goes to interest, the amount that goes to principal, and the new balance after the payment is made. A sample of the first 6 months of the amortization schedule for this loan is shown in Table 1.

By the time you finish making payments, how much interest did you pay on this loan? If you make 36 payments of $141.47, you repay your parents $5,092.92, so you have paid only $92.92 interest on this loan.

Now suppose that you go by CarMax and found the car you have always wanted – a 2000 model Porsche Boxster 2-door convertible for only $30,998. You really want this car, and you are hoping that you can afford the payments on the car loan.

Payment Number	Interest	Principal	New Balance
			$5,000.00
1	$5.00	$136.47	$4,863.53
2	$4.86	$136.61	$4,726.92
3	$4.73	$136.74	$4,590.18
4	$4.59	$136.88	$4,453.30
5	$4.45	$137.02	$4,316.28
6	$4.32	$137.15	$4,179.13

Table 1

1. State sales tax on cars in North Carolina is 2%, and you decide that you want the 5 year extended warranty for 72 months which costs 1.5% of the sales price – before taxes. How much will this package cost?

2. You have to make a 5% or greater down payment to get a loan through CarMax. How much down payment is this and how much would you have to borrow now?

3. CarMax' best rate is 4.95% for 5 years (60 monthly payments). What are the payments on this car if you borrow the maximum allowed in the previous problem?

4. Now matter how hard you try, you cannot afford more than $500 per month for car payments. How much would your down payment have to be in order to reduce the payments to $500 or less?

5. Make an amortization table for the amount borrowed in Problem 4. (Use a spreadsheet)

6. What is the total cost of the loan repayment? How much interest will you pay on this loan?

7. You ask CarMax to search their loan database and they come back with two additional offers: Wachovia at 4.85% for 50 months and Bank of America at 5% for 6 years. What is the maximum amount you can borrow under each of these scenarios if you can still pay no more than $500 per month?

8. Which of these loans is a better deal? What is the total price (including tax, warranty and interest) that you will pay for the car in each case?

Trigonometric Model

Are U.S. Cities Getting Warmer?

Purpose: Create function models based on temperature data for one of the 100 largest cities in the US.

From the latest US Census Bureau data available, select the *xy*th largest city in the US, where *x* and *y* are the last two digits of your student identification number (or Social Security number). This is your city. (For example if your student ID number ends in 26, you should find the data for the 26th largest city in the United States – which in the 2000 census would be Charlotte NC. If your ID number ends in 00, choose the 100th largest city). This information is available from the US Census Bureau at http://www.census.gov/main/www/cen2000.html.

Find the historical temperature data for your city for the years 1960, 1980, and 2000. Record the maximum and minimum temperatures for each month during those years.

1. Find the best-fitting trigonometric model for the monthly maximum temperatures for each of the years 1960, 1980, and 2000. Compare your models with respect to amplitude and average value.
2. Find the best-fitting trigonometric model for the monthly minimum temperatures for each year. Compare your models with respect to amplitude and average value.
3. Do your models provide evidence to support the theory of global warming? Why or why not?
4. Are your models in agreement with others from the same state? region? Is there one region for which it does support the theory of global warming? Is there are region for which it does not support this theory?

Piecewise-Defined Function Model

North Carolina Tax Rates

Purpose: Create function models based on tax data for the state of NC.

Find the most recent North Carolina and Federal Tax Schedules. Using the data from each, construct income tax functions for single taxpayers as a function of income.

1. Graph the functions.
2. Determine the income tax liability for taxable incomes of $20,000, $30,000, $50,000 and $100,000.
3. A person's **effective tax rate** is defined as the percent of total income that is paid in tax. What is the effective tax rate for each of the incomes in Problem 2?
4. Now, look at a single taxpayer's total tax liability as a composite function based on the two tax schedules. Create a function to describe this and plot it.
5. Now, add in the Social Security tax liability. Construct a tax function for gross income based on the most recent adjustments from the Social Security Administration. (http://www.ssa.gov/cola)