April 7, 2000

Your name ______________________________

The first 5 problems count 5 points each and, unless marked otherwise, rest count 6 points. The total value of the problems is 129.

1. Which of the following numbers belong to the *domain* of the function $h(x) = 2 - \sqrt{4 - x^2}$? Circle all those that apply.

   (A) $\boxed{\text{-2}}$ **(B)** $\boxed{0}$ **(C)** $\boxed{2}$ **(D)** 3 **(E)** 5

2. Write an equation that expresses the distance d of the origin from any point of the line $y = 3x - 5$.

   (A) $d = \sqrt{x^2 - 3x - 5}$ **(B)** $d = \sqrt{x^2 + (3x - 5)^2}$

   (C) $d = \sqrt{x^2 - (3x - 5)^2}$ **(D)** $d = \sqrt{5 + (x - 3)^2}$

   (E) $d = \sqrt{x^2 + (3x + 5)^2}$

   Solution: A typical point on the line has coordinates $(x, 3x-5)$. Therefore, $d = \sqrt{x^2 + (3x - 5)^2}$.

3. What is the degree of the polynomial defined by $p(x) = (x^2 - 2)^3 \cdot (x^4 + 3)^2$?

   (A) 8 **(B)** 10 **(C)** 12 **(D)** $\boxed{14}$ **(E)** 16

4. Note that $\sqrt{2}$ is a zero of the polynomial defined by

   $$p(x) = (x^2 - 2)^3 \cdot (x^4 + 3)^2.$$

   What is its multiplicity?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** $\boxed{3}$ **(E)** 4

5. For what value of k are the lines $2x - ky = 4$ and $3x + y = 7$ perpendicular?

   (A) -6 **(B)** -4 **(C)** 0 **(D)** $\boxed{6}$ **(E)** 8

The next few questions are short answer questions. Write your answer is the blank provided.

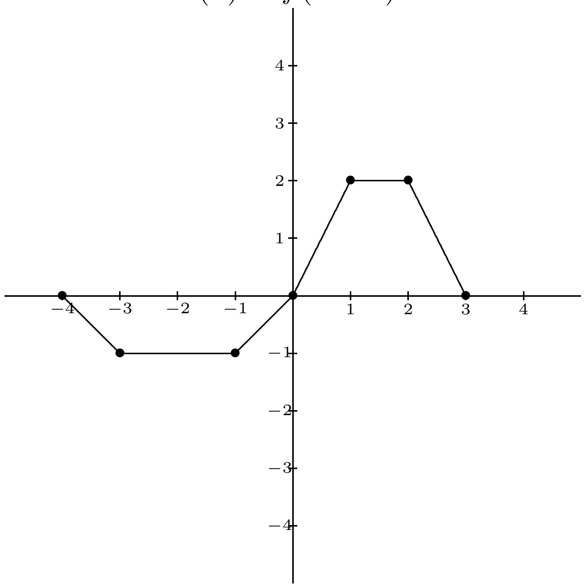
6. What is the exact value of $|7\pi - 22| + |5\pi - 15| - |7 - 2\pi|$?

   Solution: Since $7\pi - 22$ is negative, its absolute value is $22 - 7\pi$. Continuing, we have $|7\pi - 22| + |5\pi - 15| - |7 - 2\pi| = 22 - 7\pi + 5\pi - 15 - 7 + 2\pi = 0$.

7. A *median* of a triangle is a line segment connecting a vertex of the triangle with the midpoint of the side opposite the vertex. What is the length of the shortest median of the triangle with vertices $(0, 0)$, $(6, 0)$, and $(0, 8)$?

   Solution: Draw the picture to see that the median from $(0, 0)$ to $(3, 4)$ is the longest, and its length is $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

8. (10 points) Let f be the function whose graph is given below. Sketch the graph of the function h defined $h(x) = f(x - 1) - 2$.



   Solution: Move the raph one unit to the right and two units downward.

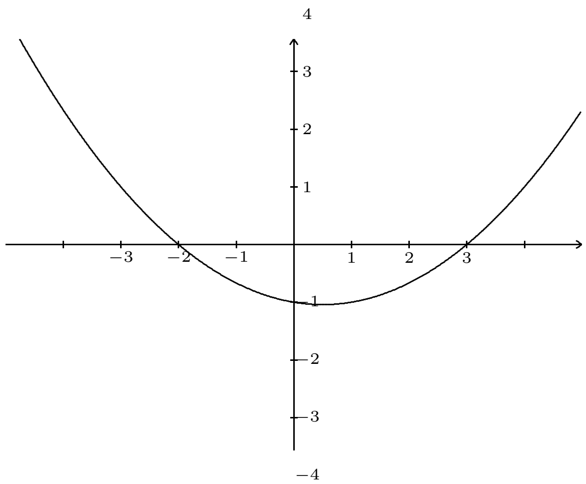
9. A topless box is built from a 12 inch by 14 inch sheet of paper by cutting from each corner a x inch by x inch square. The four squares are discarted and the flaps are folded upwards to form the sides of the box. What is the volume of the box?

   Solution: $V = x\cdot(14-2x)\cdot(12-2x)$ or $4x^3-52x^2+168x$.

10. Nick goes on a hike, leaving his home and walking 4 miles north; then he turns and walks 3 miles east, turn, and walks 5 miles south, and finally, turns again, and walks 6 miles west. How far is he from home, to the nearest tenth of a mile?

    Solution: Use a coordinate system approach to find that he goes from $(0,0)$ to $(0,4)$, then to $(3,4)$, then to $(3,-1)$, and finally to $(-3,-1)$, the distance from the origin to which is $\sqrt{3^2+1^2} = \sqrt{10}$.

11. (10 points) Find the values of a, b, and c so that the graph of $y = ax^2+bx+c$ is the one shown below. Note that the x-intercepts are -2 and 3, and the y-intercept is -1.



    Solution: First note that $f(x) = a(x+2)(x-3)$ has the desired x-intercepts. Solve the equation $f(0) = -1$ for a to get $f(x) = x^2/6-x/6-1$.

12. (20 points) Use "completing the square" to find the center and the radius of the circle given by
$$x^2 - 4x + y^2 + y = 2.$$

Solution: Add 4 to the first part and 1/4 to the y quadratic to get

$$(x-2)^2 + (y+1/2)^2 = 25/4$$

from which the center can be read off as $(2, -1/2)$ and the radius $r = 5/2$.

13. (20 points) Let

$$f(x) = \begin{cases} 2x - 3 & \text{if } x < -2 \\ x + 1 & \text{if } -2 \leq x \end{cases} \quad \text{and } g(x) = \begin{cases} |x| & \text{if } x < 4 \\ x^2 - 3 & \text{if } x \geq 4 \end{cases}$$

(a) Find and simplify a formula for the composite function $f \circ g(x)$.

Solution:

$$f \circ g(x) = \begin{cases} 2|x| - 3 & \text{if } x < 4 \text{ and } |x| < -2 \\ |x| + 1 & \text{if } x < 4 \text{ and } |x| \geq -2 \\ 2(x^2 - 3) - 3 & \text{if } x \geq 4 \text{ and } x^2 - 3 < -2 \\ x^2 - 3 + 1 & \text{if } x \geq 4 \text{ and } x^2 - 3 \geq -2 \end{cases}$$

The first and third parts are not satisfiable, so the function reduces to

$$f \circ g(x) = \begin{cases} |x| + 1 & \text{if } x < 4 \\ x^2 - 2 & \text{if } x \geq 4 \end{cases}$$

(b) Find and simplify a formula for the composite function $g \circ f(x)$.

Solution:

$$g \circ f(x) = \begin{cases} |2x - 3| & \text{if } x < -2 \text{ and } 2x - 3 < 4 \\ x + 1 & \text{if } -2 \leq x \text{ and } x + 1 < 4 \\ (2x - 3)^2 - 3 & \text{if } x < -2 \text{ and } 2x - 3 \geq 4 \\ (x + 1)^2 - 3 & \text{if } -2 \leq x \text{ and } x + 1 \geq 4 \end{cases}$$

Only the third line is unsatisfiable, so the simplified function is

$$g \circ f(x) \begin{cases} |2x - 3| & \text{if } x < -2 \\ |x + 1| & \text{if } -2 \leq x < 3 \\ x^2 + 2x - 2 & \text{if } 3 \leq x \end{cases}$$

(c) (20 points) Find an equation for each of the vertical asymptotes and of the horizontal asymptote, if one exists, of the function h defined by

$$h(x) = \frac{(x^2+2x-3)(x^2-x-6)}{(x-3)(2x-2)(x-2)^2(x+1)(x+2)^2}.$$

Solution: Factor and cancel common terms to get

$$h(x) = \frac{(x+3)}{2(x-1)(x-2)^2(x+1)(x+2)^2}.$$

Since the degree of the denominator is greater than that of the numerator, the horizontal asymptote is $y = 0$. The three vertical asymptotes are $x = 2$, $x = -1$, and $x = -2$.