

September 25, 2000

Your name \_\_\_\_\_

The first 14 problems count 6 points each and the final ones counts as marked. Problems 2 through 9 are multiple choice and 10 through 14 are fill-in-the-blank. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on problems 2 through 14.

1. Fill in the three character code you received via email in the box

2. Which of the following is a factor of  $x^4 - x$ ? Circle all those that apply.

(A)   $x$     (B)   $x - 1$     (C)   $x + 1$     (D)   $x^2 + x + 1$     (E)   $x^2 - x + 1$

3. How many roots does the equation below have?

$$x(x^2 - 3) - 4(x^2 - 3) = 0$$

(A) 0    (B) 1    (C) 2    (D)  3    (E) 4

4.

$$\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} =$$

(A)   $\frac{x+1}{x-1}$     (B)   $\frac{x-1}{x+1}$     (C)   $x - 1$     (D)   $1 - x$     (E)   $x$

5. Sandra was paid time-and-a-half for hours worked in excess of 40 per week. She earned \$416 during a week in which she worked 48 hours. What was her hourly wage?

(A) \$6.00    (B) \$6.50    (C) \$7.00    (D)  \$8.00    (E) \$8.50

6. Dick can paint a bedroom in 3 hours, Jeff can do it in 5 hours and Niki takes 6 hours. How long will it take them working together?

(A)  7/10 hours    (B)  1 hour    (C)   $1\frac{3}{7}$  hours    (D)   $2\frac{1}{3}$  hours    (E)   $4\frac{2}{3}$  hours

7. Which of the following is the sum of the two roots of  $6x^2 + 7x - 10 = 0$ ?

(A)   $-7/6$     (B)   $-1/10$     (C)   $7/10$     (D)  1.1    (E)   $2\frac{1}{6}$

8.  $(4^{-1} - 5^{-1})^{-1} =$

(A)  $-4$    (B)  $-5$    (C)  $\frac{1}{20}$    (D)  $4$    (E)  $\boxed{20}$

9. The function
- $F = \frac{9}{5}C + 32$
- can be used to find the temperature on the Fahrenheit scale when the Celsius temperature is known. Which of the following formulas can be used to find the Celsius temperature when the Fahrenheit temperature is known?

(A)  $C = \frac{9}{5}F - 32$    (B)  $C = \frac{5}{9}F - 32$    (C)  $C = \frac{9}{5}F + 32$

(D)  $C = \frac{5}{9}F + 32$    (E)  $\boxed{C = \frac{5}{9}(F - 32)}$

The next few questions are short answer questions. Write your answer in the blank provided.

10. A
- $4 \times 4 \times 4$
- cube is built from unit cubes. The entire outside surface is painted. How many of the 64 unit cubes receive some paint?

**Solution:** The number of painted cubes plus the number of unpainted cubes is 64. Therefore there are  $64 - 2^3 = 56$  unpainted cubes.

11. Simplify:
- $2x(1 - x) - 3[x + 3 - (2x - 1)]$

**Solution:**  $2x(1 - x) - 3[x + 3 - (2x - 1)] = 2x - 2x^2 - 3[x + 3 - 2x + 1]$   
 $= 2x - 2x^2 - 3[4 - x] = 2x - 2x^2 - 12 + 3x = -2x^2 + 5x - 12.$

12. The sum of two numbers is 7 and their product is 5. What is the sum of their reciprocals?

**Solution:** Let  $x$  and  $y$  be the numbers. Then  $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = 7/5.$

13. You're solving the equation
- $2x^2 - 8x + 7 = 0$
- by completing the square. After dividing by the coefficient of
- $x^2$
- , you add and subtract the same number. What is that number?

**Solution:** Divide by 2 to get  $x^2 - 4x + 7/2 = 0$ . Then add and subtract the square of half the coefficient of  $x$  to get  $x^2 - 4x + 4 - 4 + 7/2 = 0$ . Then write  $(x - 2)^2 - 1/2 = 0$ . Finally, solve this to get  $x = 2 \pm \sqrt{1/2}$ . Of course, the answer to this is just the number 4.

14. The number
- $8^4 \cdot 4^8 \cdot 8^8 \cdot 4^4$
- can be written as
- $A^B$
- , where
- $A$
- and
- $B$
- are integers (whole numbers). Write
- $A^B$
- in the blank.

**Solution:**  $8^4 \cdot 4^8 \cdot 8^8 \cdot 4^4 = 8^{12} \cdot 4^{12} = (8 \cdot 4)^{12} = 32^{12}.$

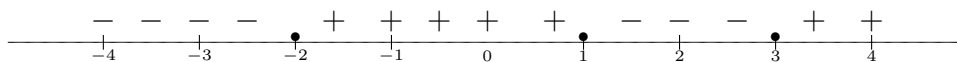
On all the following questions, **show your work**.

15. (20 points) Use the test interval technique to solve the inequality

$$\frac{(x-3)(x+2)}{x-1} \geq 0.$$

**Solution:** The critical points are  $x = 3, -2,$  and  $1,$  so we may take as test points  $x = -3, 0, 2,$  and  $4.$  The value of  $\frac{(x-3)(x+2)}{x-1}$  at these points is, respectively, negative, positive, negative, and positive. Since the inequality is a greater than **or equal to**, we must check the critical numbers themselves. Doing so we find that  $3$  and  $-2$  work but  $1$  does not. Thus we have the solution  $[-2, 1) \cup [3, \infty).$

Use the number line provided below.



16. (15 points) An open box is to be constructed from a square piece of metal by removing a square with side 1 inch from each corner and turning the sides upward. If the volume of the box must be 4 cubic inches, what should be the size of the original square?

**Solution:** Let the square have  $x$  units on each side. Then the volume is given by  $V = (x - 2)(x - 2)1$ , so we must consider the equation  $(x - 2)(x - 2)1 = 4$ . This is equivalent to  $x^2 - 4x = 0$  which has the two solutions,  $x = 0$  and  $x = 4$ . Of course, only the later is reasonable.

17. Bonus Problem (10 points). If you read my essay Philosophy of Teaching, part 1 at the website, either write a sentence about why I think it is important to read the text or name the doctor referred to in the essay, and say why he is referenced.