

September 18, 2001

Your name \_\_\_\_\_

The first 20 problems count 3 points each and the final ones counts as marked. Problems 1 through 20 are multiple choice. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on multiple choice items. You must show your work on the other problems. The total number of points available is 119.

1. Fill in the three character code you received via email in the box

2. Which of the following belongs to the domain of the function  $f(x) = \frac{\sqrt{x-7}}{(x-10)(x-11)}$ ? Circle all that apply.

(A) -2    (B) 0    (C) 5    (D) 10    (E) 14

**Solution:** E. Only 14 belongs to the domain.

3.  $\frac{10^{12} - 10^{11}}{9} =$

(A)  $\frac{1}{9}$     (B)  $\frac{10}{9}$     (C)  $10^3$     (D)  $\frac{10^{11}}{9}$     (E)  $10^{11}$

**Solution:** E. Factor to get  $\frac{10^{12} - 10^{11}}{9} = \frac{10^{11}(10 - 1)}{9} = 10^{11}$

4. If  $x \geq 0$ , then  $\sqrt{x\sqrt{x\sqrt{x}}}$  =

(A)  $x\sqrt{x}$     (B)  $x^4\sqrt{x}$     (C)  $\sqrt[8]{x}$     (D)  $\sqrt[8]{x^3}$     (E)  $\sqrt[8]{x^7}$

**Solution:** E.  $\sqrt{x\sqrt{x\sqrt{x}}} = \sqrt{x \cdot \sqrt{x^{3/2}}} = \sqrt{x \cdot x^{3/4}} = \sqrt{x^{7/4}} = x^{7/8}$ .

5. If  $\sqrt{2 + \sqrt{x}} = 3$ , then  $x =$

(A) 1    (B) 7    (C) 11    (D) 49    (E) 121

**Solution:** D. Square both sides to get  $2 + \sqrt{x} = 9$  which is equivalent to  $x = 49$ .

6. If  $xy \neq 0$ , which of the following must be true?

- (A) Either  $x > 0$  or  $y > 0$     (B) Either  $x$  or  $y$  is an integer  
(C) Neither  $x$  nor  $y$  is 0    (D) Either  $x$  or  $y$  is 0  
(E) Both  $x$  and  $y$  are 0

**Solution:** C. In order that  $xy \neq 0$ , it is necessary that both  $x$  and  $y$  be nonzero.

7. What is the remainder when  $x^2 + 3x - 5$  is divided by  $x - 1$ ?

- (A)  $-5$     (B)  $-2$     (C)  $-1$     (D)  $0$     (E)  $1$

**Solution:** C. The remainder is the same as the value of  $f(x) = x^2 + 3x - 5$  at  $x = 1$  which is  $f(1) = -1$ .

8. If  $x^2 = y^2$ , then which one of the following must be true?

- (A)  $x = y$     (B)  $x = -y$     (C)  $x^3 = y^3$     (D)  $|x| = |y|$     (E)  $\sqrt{x} = \sqrt{y}$

**Solution:** D. Either  $y = x$  or  $y = -x$ , which is equivalent to  $|x| = |y|$ .

9. Tom is three years older than Sue. The sum of their ages is 15. Given that Sue's age is  $x$  years, which of the equations could be solved to find  $x$ ?

- (A)  $x = 15 - 3$     (B)  $x + (x - 3) = 15$     (C)  $x + 3x = 15$   
(D)  $x + (x + 3) = 15$     (E)  $x = 15 + (x - 3)$

**Solution:** D. The equation is  $x + (x + 3) = 15$ .

10. What is the exact value of  $|\sqrt{2} - 2| + |3\sqrt{2} - 5|$ ?

- (A)  $4\sqrt{2} - 7$     (B)  $4\sqrt{2} + 7$     (C)  $7 - 4\sqrt{2}$     (D)  $1.34$     (E)  $3 - 2\sqrt{2}$

**Solution:**  $|\sqrt{2} - 2| + |3\sqrt{2} - 5| = 2 - \sqrt{2} + 5 - 3\sqrt{2} = 7 - 4\sqrt{2}$

11. If  $f(x + 1) = x^4 - x + 1$ , what is  $f(0)$ ?

- (A)  $0$     (B)  $1$     (C)  $2$     (D)  $3$     (E)  $4$

**Solution:** D.  $f(0) = f(-1 + 1) = (-1)^4 - (-1) + 1 = 3$ .

12. The repeating decimal  $0.24242424\dots = 0.\overline{24}$  has the value  $m/n$  where  $m/n$  is reduced to lowest terms and  $m$  and  $n$  are integers. What is  $m + n$ ?
- (A) 36    (B) 41    (C) 42    (D) 46    (E) 120

**Solution:** B. Let  $x$  denote  $0.24242424\dots$ . Then  $100x = 24.24242424\dots$  so  $100x - x = 99x = 24$  and it follows that  $33x = 8$ , so  $n = 33$  and  $m = 8$ .

13. When  $\frac{4}{x-2} - \frac{x}{x-2}$  is expressed as a simple fraction, the numerator could be
- (A) 4    (B)  $x$     (C)  $4 - x$     (D)  $4 + x$     (E)  $x - 2$

**Solution:** C. Note that  $\frac{4}{x-2} - \frac{x}{x-2} = \frac{4-x}{x-2}$ .

14. What is the  $y$ -intercept of the line  $L$  satisfying
- (a)  $L$  is parallel to the line defined by  $4x - 2y = 3$ , and
- (b)  $L$  contains the point  $(-2, -1)$ .
- (A)  $-3$     (B)  $-2$     (C)  $-3/2$     (D)  $-1$     (E)  $3$

**Solution:** E. The line has slope  $4/2 = 2$  and has an equation given by  $y + 1 = 2(x + 2)$  which is equivalent to  $y = 2x + 3$ .

15. Which of the following is a factor of  $x^4 - x$ ? Circle all those that apply.
- (A)  $x$     (B)  $x - 1$     (C)  $x + 1$     (D)  $x^2 + x + 1$     (E)  $x^2 - x + 1$

**Solution:** A, B, and D. Note that  $x^4 - x = x(x^3 - 1) = x(x - 1)(x^2 + x + 1)$ .

16. How many roots does the equation below have?

$$x(x^2 - 3) - 4(x^2 - 3) = 0$$

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

**Solution:** Factor to get  $(x - 4)(x^2 - 3) = 0$ . The quadratic factor has two zeros, so there are three roots altogether.

17.

$$\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} =$$

- (A)  $\frac{x+1}{x-1}$     (B)  $\frac{x-1}{x+1}$     (C)  $x-1$     (D)  $1-x$     (E)  $x$

**Solution:** A. Note that  $\frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \frac{x+1}{x-1}$ .

18. Niki was paid time-and-a-half for hours worked in excess of 40 per week. She earned \$416 during a week in which she worked 48 hours. What was her hourly wage?

- (A) \$6.00    (B) \$6.50    (C) \$7.00    (D) \$8.00    (E) \$8.50

**Solution:** D. Niki was paid for  $40 + (3/2)8 = 52$  hours of work, so her rate of pay is  $416 \div 52 = 8$  dollars per hour.

19. Which of the following is the sum of the two roots of  $6x^2 + 7x - 10 = 0$ ?

- (A)  $-7/6$     (B)  $-1/10$     (C)  $7/10$     (D)  $1.1$     (E)  $2\frac{1}{6}$

**Solution:** A. Factor the quadratic to get  $6x^2 + 7x - 10 = (6x - 5)(x + 2) = 0$ , the add the zeros to get  $5/6 - 2 = -7/6$ .

20.  $(5^{-1} - 4^{-1})^{-1} =$

- (A)  $-20$     (B)  $-4$     (C)  $1$     (D)  $\frac{1}{20}$     (E)  $20$

**Solution:** A.  $(1/5 - 1/4)^{-1} = -20$ .

On all the following questions, **show your work.**

21. (5 points) Simplify:  $4x(1-x) - 3[x+3 - (2x-1)]$

**Solution:**  $4x(1-x) - 3[x+3 - (2x-1)] = 4x - 4x^2 - 3[x+3 - 2x+1]$   
 $= 4x - 4x^2 - 3[4-x] = 4x - 4x^2 - 12 + 3x = -4x^2 + 7x - 12$ .

22. (7 points) The points  $A = (0, 0)$ ,  $B = (8, 0)$ , and  $C = (3, 7)$  are the vertices of triangle. Find the length of the longest side.

**Solution:** The distances are  $d_1 = \sqrt{8^2}$ ,  $d_2 = \sqrt{3^2 + 7^2} = \sqrt{58}$ , and  $d_3 = \sqrt{5^2 + 7^2} = \sqrt{74}$ , the largest of which is  $d_3 = \sqrt{74}$ .

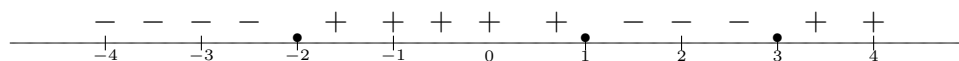
23. (12 points) Solve the equation  $2x^2 - 8x + 7 = 0$  by completing the square.

**Solution:** Divide by 2 to get  $x^2 - 4x + 7/2 = 0$ . Then add and subtract the square of half the coefficient of  $x$  to get  $x^2 - 4x + 4 - 4 + 7/2 = 0$ . Then write  $(x - 2)^2 - 1/2 = 0$ . Finally, solve this to get  $x = 2 \pm \sqrt{1/2}$ . Of course, the answer to this is just the number 4.

24. (20 points) Use the test interval technique to solve the inequality

$$(x - 3)(x + 2)(x - 1) \geq 0.$$

**Solution:** The critical points are  $x = 3, -2$ , and  $1$ , so we may take as test points  $x = -3, 0, 2$ , and  $4$ . The value of  $(x - 3)(x + 2)(x - 1)$  at these points is, respectively, negative, positive, negative, and positive. Since the inequality is a greater than **or equal to**, we must check the critical numbers themselves. Doing so we find that  $3, -2$  and  $1$  all work. Thus we have the solution  $[-2, 1] \cup [3, \infty)$ .



25. (15 points) Solve the inequality  $|2x - 5| \geq 9$ .

**Solution:** Either  $2x - 5 \geq 9$  or  $2x - 5 \leq -9$ . The former is true if  $2x \geq 14$  or  $x \geq 7$ . The later is true if  $2x \leq -4$  or  $x \leq -2$ . So the solution set is  $(-\infty, -2] \cup [7, \infty)$ . Alternatively, use the Test Interval Technique, critical points  $-2$  and  $7$ , etc.