

September 18, 2002

Your name \_\_\_\_\_

The first 11 problems count 5 points each and the final ones counts as marked. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on multiple choice items. You must show your work on the other problems. The total number of points available is 131.

1. Fill in the three character code you received via email in the box

2.  $\frac{9^{12} - 9^{11}}{8} =$

(A)  $\frac{1}{8}$     (B)  $\frac{9}{8}$     (C)  $9^3$     (D)  $\frac{9^{11}}{8}$     (E)  $9^{11}$

(E)  $\frac{9^{12} - 9^{11}}{8} = \frac{9^{11}(9-1)}{8} = 9^{11}$ .

3. If  $\sqrt{2 + \sqrt{x}} = 3$ , then  $x =$

(A) 1    (B) 7    (C) 11    (D) 49    (E) 121

(D) Square both sides to get  $2 + \sqrt{x} = 9$  from which it follows that  $\sqrt{x} = 7$  and  $x = 49$ .

4. Tom is three years older than Sue. The sum of their ages is 15. Given that Tom's age is  $x$  years, which of the equations could be solved to find  $x$ ?

(A)  $x = 15 - 3$     (B)  $x + (x - 3) = 15$     (C)  $x + 3x = 15$

(D)  $x + (x + 3) = 15$     (E)  $x = 15 + (x - 3)$

(B) If Tom is  $x$  years old, then Sue is  $x - 3$  years old. Thus the equation is  $x + (x - 3) = 15$ .

5. What is the exact value of  $|\sqrt{3} - 2| + |3\sqrt{3} - 6|$ ?

(A)  $4\sqrt{3} - 8$     (B)  $4\sqrt{3} + 8$     (C)  $8 - 4\sqrt{3}$     (D) 1.07    (E)  $4 - 2\sqrt{3}$

(C) Recall from the definition, that  $|\sqrt{3} - 2| = 2 - \sqrt{3}$  while  $|3\sqrt{3} - 6| = 6 - 3\sqrt{3}$ , so their sum is  $8 - 4\sqrt{3}$ .

6.  $3^3 \cdot 6^3 \cdot 3^6 \cdot 6^6 =$

(A)  $9^9$     (B)  $9^{18}$     (C)  $18^9$     (D)  $18^{18}$     (E)  $324^{18}$

(C) Note that  $3^3 \cdot 6^3 \cdot 3^6 \cdot 6^6 = 3^3 \cdot 3^6 \cdot 6^3 \cdot 6^6 = 3^9 \cdot 6^9 = 18^9$ .

7. What is the product of the roots of  $(x - 1)(x - 2) + (x - 2)(x - 5) = 0$ ?

- (A) 2    (B) 6    (C) 10    (D) 12    (E) 20

(B) Factor the  $(x - 2)$  to get  $(x - 2)[(x - 1) + (x - 5)] = (x - 2)(2x - 6) = 0$ , so the roots are  $x - 2$  and  $x = 3$ .

8. Which of the following is a factor of  $x^4 + x$ ? Circle all those that apply.

- (A)  $x$     (B)  $x - 1$     (C)  $x + 1$     (D)  $x^2 + x + 1$     (E)  $x^2 - x + 1$

(A,C,E) The expression  $x^4 + x$  factors as follows:  $x^4 + x = x(x^3 + 1) = x(x + 1)(x^2 - x + 1)$ .

9. What is the sum of the two roots of  $2x^2 - 13x + 15 = 0$ ?

- (A)  $3/2$     (B) 5    (C) 6.5    (D) 7    (E) 9.5

(C) Factor to get roots of  $x = 3/2$  and  $x = 5$ .

10. How many roots does the equation below have?

$$x^2(x^2 - 3) - 4(x^2 - 3) = 0$$

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

11.

$$\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} =$$

(E) The equation has four roots,  $x = \pm 2$ ,  $x = \sqrt{3}$ , and  $x = -\sqrt{3}$ .

- (A)  $\frac{x + 1}{x - 1}$     (B)  $\frac{x - 1}{x + 1}$     (C)  $x - 1$     (D)  $1 - x$     (E)  $x$

(A) Find a common denominator for each of the two fractions, and then invert and multiply.

On all the following questions, **show your work**.

12. (10 points) Find the exact value of  $(6^{-1} - 5^{-1})^{-1}$ . Show your work. Calculator solutions are not acceptable.

**Solution:**  $(6^{-1} - 5^{-1})^{-1} = (\frac{1}{6} - \frac{1}{5})^{-1} = (-\frac{1}{30})^{-1} = -30$ .

13. (20 points) Use the test interval technique to solve the inequality

$$(3x - 4)(2x + 3)(x - 4) \geq 0.$$

Use the number line provided below. Express your answer using interval notation.

**Solution:** The critical points are  $x = 4, -3/2,$  and  $4/3,$  so we may take as test points  $x = -2, 0, 2,$  and  $5.$  The value of  $(3x - 4)(2x + 3)(x - 4)$  at these points is, respectively, negative, positive, negative, and positive. Since the inequality is a greater than **or equal to**, we must check the critical numbers themselves. Doing so we find that  $4, -3/2$  and  $4/3$  all work. Thus we have the solution  $[-3/2, 4/3] \cup [4, \infty).$

14. (10 points) Niki runs at a rate of 800 ft. per minute. Jeff runs at the rate of 900 ft. per minute. If Niki starts 500 feet ahead of Jeff, how long does it take Jeff to catch up with her.

**Solution:** Jeff gains 100 feet per minute, so it takes him 5 minutes to catch up.

15. (12 points) Solve the equation  $3 + \sqrt{3x + 1} = x.$

**Solution:** Write  $\sqrt{3x + 1} = x - 3,$  square both sides to get  $3x + 1 = x^2 - 6x + 9,$  which is equivalent to  $x^2 - 9x + 8 = 0,$  which has two roots,  $x = 1$  and  $x = 8.$  However,  $x = 1$  is extraneous.

16. (12 points) Solve the equation  $2x^2 - 8x + 7 = 0$  by completing the square.

**Solution:** Divide by 2 to get  $x^2 - 4x + 7/2 = 0.$  Then add and subtract the square of half the coefficient of  $x$  to get  $x^2 - 4x + 4 - 4 + 7/2 = 0.$  Then write  $(x - 2)^2 = 1/2.$  Finally, solve this to get  $x = 2 \pm \sqrt{1/2}.$

17. (12 points) Solve the inequality  $|2x - 5| \geq 9.$

**Solution:** Solve the two inequalities,  $2x - 5 \geq 9$  and  $2x - 5 \leq -9$  to get  $x \geq 7$  OR  $x \leq -2,$  which we can write as  $(-\infty, -2] \cup [7, \infty).$