

September 25, 1998

Your name _____

The first 12 problems count 6 points each and the final 3 count 15 points each.

1. Fill in the three character code you received via email in the box

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

2. Simplify $\left(\frac{2x}{y^2}\right)^{-3}$
- (A) $-\frac{8y^6}{x^3}$ (B) $\frac{y^6}{8x^3}$ (C) $\frac{y^5}{8x^3}$ (D) $\frac{y^6}{6x^3}$ (E) none of A, B, C or D

The answer is B.

2. Evaluate: $\left(-\frac{64}{27}\right)^{-2/3}$
- (A) $\frac{9}{16}$ (B) $-\frac{9}{16}$ (C) $\frac{16}{9}$ (D) $-\frac{16}{9}$ (E) $\frac{128}{81}$

The answer is A.

3. Simplify: $x - 2[3 - (x - 1) + x]$
- (A) $-3x - 4$ (B) $5x - 8$ (C) $x - 8$ (D) $x - 4$
- (E) none of A, B, C or D

The answer is C.

4. $3^3 \cdot 6^3 \cdot 3^6 \cdot 6^6 =$
- (A) 9^9 (B) 9^{18} (C) 18^9 (D) 18^{18} (E) 324^{18}

The answer is C.

5. $(6^{-1} - 5^{-1})^{-1} =$
- (A) -30 (B) -1 (C) $\frac{1}{30}$ (D) 1 (E) 30

The answer is A.

6. The equation $x^2 - 2x + y^2 + 4y = 20$ describes a circle with center at (h, k) and radius r . Find $h + k + r$.

(A) 4 (B) 5 (C) 8 (D) 9 (E) 24

The answer is A.

6. The equation $x^2 - 2x + y^2 + 4y = 31$ describes a circle with center at (h, k) and radius r . Find $h + k + r$.

(A) 4 (B) 5 (C) 8 (D) 9 (E) 24

The answer is B.

7. Let A be the point $(1, 2)$. Let B be the result of reflecting A through the y -axis. Let C be the reflection of B through the line $y = x$, and let D be the result of reflecting C through the x -axis. Find the distance between A and D .

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) 2 (E) $3\sqrt{2}$

The answer is C.

7. The point $P = (6, -3)$ is reflected about the y -axis to a point Q . Then Q is reflected about the x -axis to get a point R . Then R is reflected about the origin to get point T . Then $T =$

(A) $(6, 3)$ (B) $(6, -3)$ (C) $(-6, 3)$ (D) $(-6, -3)$ (E) $(-3, 6)$

The answer is B.

8. Mark McGuire has just hit another home run. This one went into the upper-deck of the ball park. The base of the upper deck is 360 feet from home plate and 150 feet high. Approximately how far from home plate did the ball travel?

(A) 370 (B) 380 (C) 390 (D) 400 (E) 410

The answer is C.

8. Erica went on a long hike. She walked three miles north, then three miles east, then one mile north, and finally six miles west. How far was she from her starting point? (Assume the earth is flat, and her hike takes place in North Carolina.)

(A) $\sqrt{5}$ (B) $\sqrt{11}$ (C) $2\sqrt{3}$ (D) $3\sqrt{5}$ (E) 5

The answer is E.

9. Suppose a is a real number such that $(a - 2)(a^2 + 3) = 0$. What is the value of $a^{a+1} + (a + 1)^a$?

(A) 7 (B) 9 (C) 11 (D) 13 (E) 17

The answer is E.

10. The function $F = \frac{9}{5}C + 32$ can be used to find the temperature on the Fahrenheit scale when the Celsius temperature is known. Which of the following formulas can be used to find the Celsius temperature when the Fahrenheit temperature is known?

(A) $C = \frac{9}{5}F - 32$ (B) $C = \frac{5}{9}F - 32$ (C) $C = \frac{9}{5}F + 32$
 (D) $C = \frac{5}{9}F + 32$ (E) $C = \frac{5}{9}(F - 32)$

The answer is E.

11. What is the exact value of $|3 - \pi|$?

(A) $3 + \pi$ (B) $3 - \pi$ (C) $\pi - 3$ (D) $1/7$ (E) 0.1415

The answer is C.

11. What is the exact value of $|\sqrt{2} - 2|$?

(A) 0.5857 (B) -0.5857 (C) $2 - \sqrt{2}$ (D) $2 + \sqrt{2}$ (E) $\sqrt{2} - 2$

The answer is C.

12. What is the product of the roots of $(x - 1)(x - 2) + (x - 2)(x - 7) = 0$?

(A) 2 (B) 6 (C) 8 (D) 12 (E) 28

The answer is C.

12. What is the product of the roots of $(x - 1)(x - 2) + (x - 2)(x - 9) = 0$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 28

The answer is D.

On all the following questions, **show your work.**

13. Find a point on the x -axis that is equidistant from $(10, 0)$ and $(0, 20)$. Solution. The point must be on the x -axis, so must be of the form $(a, 0)$. It must satisfy the equation

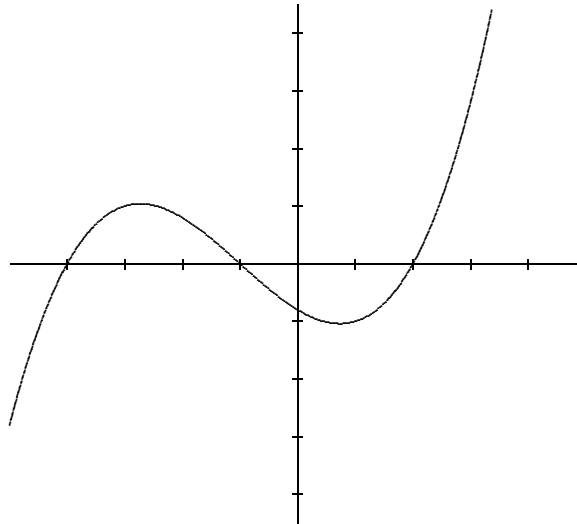
$$\sqrt{(10 - a)^2 + (0 - 0)^2} = \sqrt{(a - 0)^2 + (0 - 20)^2}.$$

From this it follows that $100 - 20a + a^2 = a^2 + 20^2$, and from this it follows that $-20a = 300$. Thus the point we seek is $(-15, 0)$.

13. Find a point on the x -axis that is equidistant from $(12, 0)$ and $(0, 22)$.
 Answer: $(a, 0) = (-\frac{340}{24}, 0) = (-14\frac{1}{6}, 0)$.

14. Find all x satisfying the inequality $(x - 2)(x + 2)(x + 4) < 0$.
 Answer: Test the intervals $(-\infty, -4)$, $(-4, -2)$, $(-2, 2)$, $(2, \infty)$ to find that the product in question is negative on the first and third of the intervals, ie, $(-\infty, -4) \cup (-2, 2)$.

15. Find all x satisfying the inequality $(x - 2)(x + 1)(x + 4) < 0$. Solution. The critical points are $x = 2$, $x = -1$, and $x = -4$. Thus there are four test intervals: $(-\infty, -4)$, $(-4, -1)$, $(-1, 2)$, $(2, \infty)$. Examine one point in each interval to find that the shape is the same as the function below.



15. Explain how you can describe the graph of the quadratic equation

$$y = ax^2 + bx + c$$

based on the coefficients a, b , and c . In particular, address the questions (a) does the curve open upwards or downwards, and (b) does it have x -intercepts? Solution. Let $D = b^2 - 4ac$. There are six possible shapes for the parabola, three of which are shown with the corresponding values of a and D .

