

October 18, 2001

Your name _____

The first 10 problems count 6 points each and the final ones counts as marked. Problems 1 through 10 are multiple choice. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on multiple choice items. You must show your work on the other problems. The total number of points available is 117.

1. Which of the following is the exact value of $|6 - 2\pi| + |8\pi - 25|$?

(A) $10\pi - 19$ (B) $10\pi - 31$ (C) $10\pi + 31$ (D) $6\pi - 19$ (E) $6\pi + 19$

Solution: B. First note that $|6 - 2\pi| = 2\pi - 6$ because $6 - 2\pi < 0$. Then note that $|8\pi - 25| = 8\pi - 25$ because $8\pi - 25 > 0$. So the sum is $2\pi - 6 + 8\pi - 25 = 10\pi - 31$.

2. The points $(-1, 3)$, $(2, 4)$, and $(x, 10)$ belong to the line L for which of the following values of x ?

(A) 18 (B) 19 (C) 20 (D) 21 (E) 22

Solution: C. The slope of the line is $1/3$ so the number x satisfies $1/3 = (10 - 4)/(x - 2)$. Thus $x = 20$.

3. What is the length of the hypotenuse of the right triangle whose vertices are at the points $(0, 1)$, $(3, 1)$, and $(3, -3)$?

(A) $2\sqrt{2}$ (B) $3\sqrt{2}$ (C) 5 (D) $4\sqrt{2}$ (E) 6

Solution: C. The legs have lengths 3, 4, and $\sqrt{3^2 + 4^2} = 5$, so the hypotenuse must have length 5.

4. The equation $x^2 - 8x + y^2 + 10y = 8$ describes a circle with center at (h, k) and radius r . Find $h + k + r$.

(A) 4 (B) 6 (C) 10 (D) 14 (E) 20

Solution: B. The completed square form is $x^2 - 8x + 16 + y^2 + 10y + 25 = 8 + 16 + 25 = 49$ so $h = 4, k = -5$, and $r = 7$. Thus $h + k + r = 6$.

5. Suppose the value of a new car declines linearly over a ten year period from the original value of \$20,000 to the value \$2,000. What is the value of the car after six years?

(A) \$8,800 (B) \$9,200 (C) \$11,000 (D) \$12,800 (E) \$14,600

Solution: B. The car loses $(20,000 - 2,000) \div 10 = 1,800$ every year. So after 6 years, it is worth $20,000 - 6 \cdot 1,800 = 9,200$.

6. Yasmin starts at a point A and walks north 2 miles, then walks east 3 miles, then south 1 mile, then east 1 mile, and finally she walks north again 2 miles, arriving at point B . Which of the following is closest to the distance between point A and point B ?

(A) 3 (B) 4 (C) 5 (D) 5.8 (E) 6.3

Solution: C. Use coordinates to keep track of where she is: $(0, 0)$ to $(0, 2)$ to $(3, 2)$ to $(3, 1)$ to $(3, 4)$ which is $\sqrt{3^2 + 4^2} = 5$ miles from the origin.

Suppose the functions f and g are given completely by the table of values shown. The next three problems require the use of the table.

x	$f(x)$	$g(x)$
0	2	5
1	7	7
2	6	4
3	1	2
4	3	6
5	6	3
6	0	1
7	4	3

7. What is $g(g(2) + f(0) - g(3))$?

(A) 1 (B) 3 (C) 4 (D) 5 (E) 6

Solution: E. Note that $g(2) + f(0) - g(3) = 4 + 2 - 2 = 4$ and $g(4) = 6$.

8. Which of the follow values of x satisfies $f(g(f(x))) = 0$?

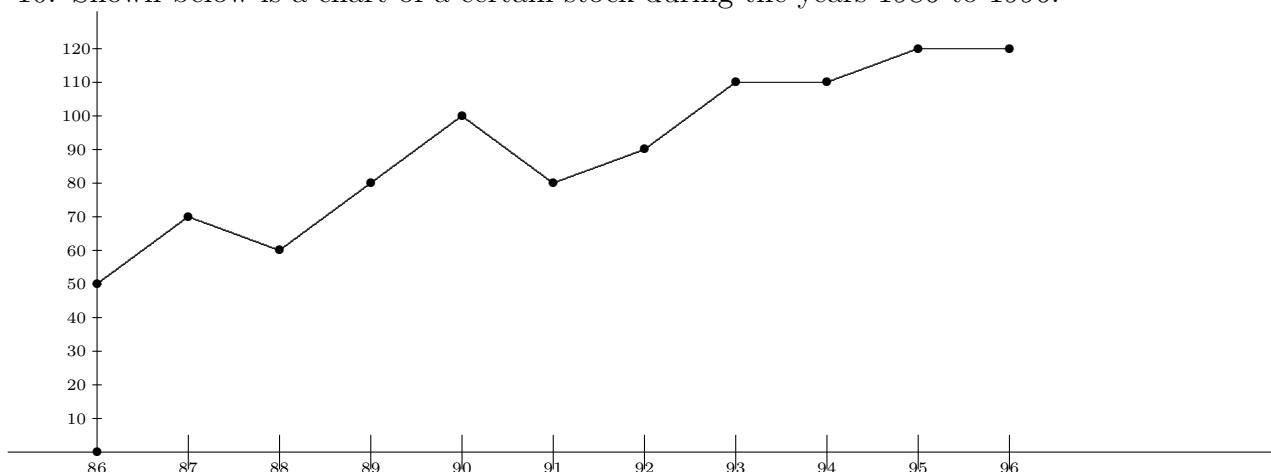
(A) 0 (B) 1 (C) 4 (D) 6 (E) 7

Solution: E. Note that $f(6) = 0$ so $g(f(x)) = 6$. Then note that $g(4) = 6$, so we need to find an x such that $f(x) = 4$. Only $x = 7$ works.

9. What is $g(g(3) \cdot g(5) - f(7))$?
- (A) 1 (B) 3 (C) 4 (D) 5 (E) 6

Solution: C. Note that $g(3) \cdot g(5) - f(7) = 2 \cdot 3 - 4 = 2$, so $g(g(3) \cdot g(5) - f(7)) = g(2) = 4$.

10. Shown below is a chart of a certain stock during the years 1986 to 1996.



By what percentage increase did the stock grow during the two years between 1988 and 1990?

- (A) 40% (B) 50% (C) 60% (D) 66% (E) 100%

Solution: D. We need to measure the gain relative to the initial value. This is given by $\frac{100-60}{60} = 2/3 = 66.6\%$ On all the following questions, **show your work.**

11. (10 points) Let $h(x) = \sqrt{5 - x^2}$. Find two functions f and g , both simpler than h such that $f \circ g(x) = h(x)$.

Solution: There are two solutions. A. Let $g(x) = 5 - x^2$ and $f(x) = \sqrt{x}$, or B. let $g(x) = x^2$ and $f(x) = \sqrt{5 - x}$.

12. (10 points) Let $g(x) = 2x - 3$ and

$$f(x) = \begin{cases} |2x| - x^2 & \text{if } x \leq 2 \\ 3x - 5 & \text{if } x > 2 \end{cases}$$

- (a) Compute the values $g \circ f(0)$, $g \circ f(1)$, $g \circ f(2)$, $g \circ f(3)$, and $g \circ f(\pi)$.

Solution: $g \circ f(0) = g(0) = -3$, $g \circ f(1) = g(1) = -1$, $g \circ f(2) = g(0) = -3$, $g \circ f(3) = g(4) = 5$, and $g \circ f(\pi) = g(3\pi - 5) = 2(3\pi - 5) - 3 = 6\pi - 13$.

- (b) Find a (symbolic) representation of $g \circ f$.

Solution:

$$g \circ f(x) = \begin{cases} 2(|2x| - x^2) - 3 & \text{if } x \leq 2 \\ 2(3x - 5) - 3 & \text{if } x > 2 \end{cases}$$

which simplifies to

$$g \circ f(x) = \begin{cases} 4|x| - 2x^2 - 3 & \text{if } x \leq 2 \\ 6x - 13 & \text{if } x > 2 \end{cases}$$

13. (12 points) Suppose $g(x) = x^2$ on the domain $[-2, 1]$. Sketch the graph of each of the following functions and state the domain of each.

- (a) $f(x) = g(2x)$

Solution: Since g is defined only when $-2 \leq x \leq 1$, it follows that f is defined when $-2 \leq 2x \leq 1$. IE, $-1 \leq x \leq 0.5$. The graph is the part of $y = 4x^2$ over the interval $[-1, 0.5]$.

- (b) $h(x) = 3 - g(x - 1)$

Solution: The replacement of x by $x - 1$ has the effect of pushing the graph to the right 1 unit. The $-$ sign flips the graph about the x -axis, and the (addition of) 3 moves that flipped graph up by 3 units. Alternatively, it the graph of $h(x) = 3 - g(x - 1) = 3 - (x - 1)^2 = 3 - x^2 + 2x + 2$ that lies over the interval $[-1, 2]$.

- (c) $k(x) = 4g(x + 1)$

Solution: Translate the graph $y = x^2$ one unit to the left, then stretch the graph by a factor of 4.

14. (10 points) Find an equation for the line L satisfying

- (a) L is parallel to the line defined by $4x - 2y = 3$, and
(b) L contains the point $(3, 1)$.

Solution: The slope must be the same as the given line, which is 2. The line L passes through $(3, 1)$, so it must satisfy $y - 1 = 2(x - 3)$ and this is just $y = 2x - 5$.

15. (15 points) Explain how you can describe the graph of the quadratic equation

$$y = ax^2 + bx + c$$

based on the coefficients a, b , and c . Hint: it may be useful to define the discriminant D to be $b^2 - 4ac$. In particular, address the questions (a) does the curve open upwards or downwards, and (b) does it have x -intercepts? Your job on this problem is to tell someone how they can easily find out about the nature of the parabola.

Solution: You don't know a, b , and c , so the best you can do is give a method for finding out based on these three numbers. Compute $D = b^2 - 4ac$. If $a > 0$, the parabola opens upward, and if $a < 0$ it opens downward. If $D > 0$ there are two intercepts, if $D = 0$, there is just one intercept, and if $D < 0$, there are no intercepts.