

October 16, 1998

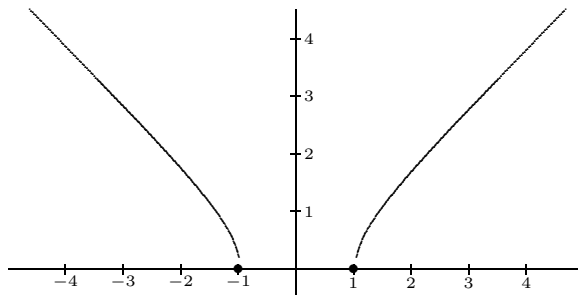
Your name _____

The first 13 problems count 6 points each and the final 2 count 20 points each.

1. Which of the following numbers belong to the *range* of the function $h(x) = 2 + \sqrt{16 - x^2}$? Circle all those that apply.

(A) -2 (B) 0 (C) 2 (D) 3 (E) 5

2. Which of the following numbers belong to the *domain* of the function shown below? Again circle all those that apply.

(A) -2 (B) 0 (C) 1/2 (D) 1 (E) 4

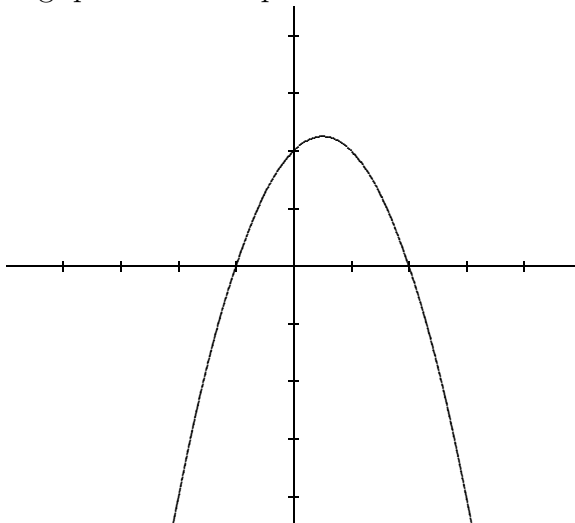
3. The function $g(x) = x^2 - 4$ can be classified as

(A) odd (B) even (C) neither odd nor even

4. Suppose $(u + v)^2 = 4$ and $(u - v)^2 = 36$. What is $u^2 + v^2$?

(A) 12 (B) 20 (C) 24 (D) 40 (E) 80

5. The graph of $y = ax^2 + bx + c$ is shown for certain values of a, b , and c . Which of the following quantities are positive? Circle all that apply.



- (A) a (B) c (C) $b^2 - 4ac$ (D) $9a + 3b + c$
6. Suppose g is defined by $g(x) = \frac{4-x}{6}$. Let f be the inverse of the function g . Then $f(2) =$
- (A) -16 (B) -8 (C) $1/3$ (D) 2 (E) 8
7. The quotient $(9x^3 - 18x^2 - 16x + 32) \div (x - 2)$ is a polynomial of the form $ax^2 + bx + c$. What is $a + b + c$?
- (A) -7 (B) -2 (C) 5 (D) 7 (E) 16
8. Suppose p is a polynomial function some of whose values are given in the table. How many x -intercepts are guaranteed by the Intermediate Value Theorem?

x	$p(x)$
-4	2
-3	1
-2	-1
-1	-2
0	3
1	5
2	-1
3	-4

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

9. Find the sum of the zeros of the cubic function $f(x) = x^3 - 13x^2 + 36x$.

- (A) -13 (B) -7 (C) 0 (D) 7 (E) $\boxed{13}$

10.-13. Let

$$f(x) = \begin{cases} 2x - 3 & \text{if } x < -2 \\ x + 5 & \text{if } -2 \leq x < 2 \\ 6 - x & \text{if } 2 \leq x \end{cases} \quad \text{and} \quad g(x) = \begin{cases} |x| & \text{if } x < 4 \\ x^2 - 1 & \text{if } x \geq 4 \end{cases}$$

10. What is $g \circ f(2)$?

- (A) 1 (B) 4 (C) 7 (D) $\boxed{15}$ (E) 48

11. Again referring to the f defined above, find a value of x for which $f(x) = 0$.

- (A) -6 (B) -5 (C) 0 (D) $3/2$ (E) $\boxed{6}$

12. Again referring to the f and the g defined above, suppose $0 < x < 1$. Which of the expressions describes the value of $g(f(x))$?

- (A) $|x + 5|$ (B) $|2x - 3|$ (C) $\boxed{(x + 5)^2 - 1}$ (D) $|6 - x|$ (E) 35

13. Again using the functions defined above, compute $(f/g)(-1) + (f \cdot g)(3)$.

- (A) 4 (B) 9 (C) $\boxed{13}$ (D) 17 (E) 22

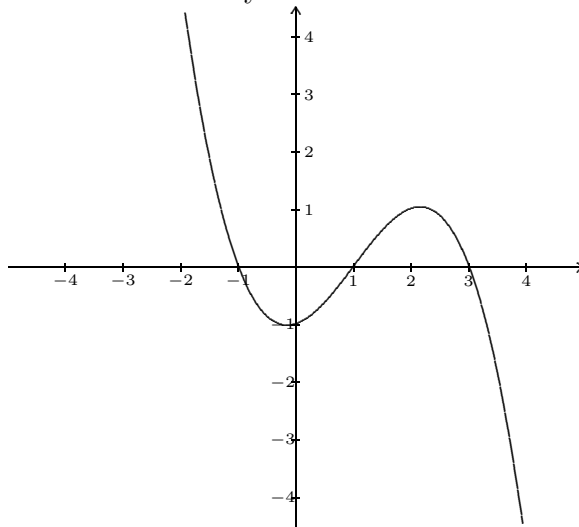
On all the following questions, **show your work**.

14. Use the ‘completing the square’ technique to find the standard form of the quadratic function $y = x^2 + bx + c$ in terms of b and c . Then find the vertex of the parabola.

$$\begin{aligned}y &= x^2 + bx + c \\&= x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \\&= \left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b^2}{4}\right) \\&= (x - h)^2 + k.\end{aligned}$$

It follows that the vertex of the parabola is at $\left(-\frac{b}{2}, c - \frac{b^2}{4}\right)$.

15. Find a symbolic representation of the cubic polynomial whose graph is given below. Remember to show all your work.



The polynomial has three intercepts, which can arise only from having certain linear factors. Remember from the factor theorem that r is an intercept of polynomial $f(x)$ if and only if $x - r$ is a factor of the polynomial. Therefore the polynomial has the form

$$f(x) = a(x + 1)(x - 1)(x - 3),$$

because the intercepts are $-1, 1,$ and 3 . To find a , note that $f(0) = -1$ and that $f(0) = a(0 + 1)(0 - 1)(0 - 3) = 3a$. Thus $a = -\frac{1}{3}$.