

April 2000

Your name _____

Problems count 5 points each.

1. Find equations for all vertical and horizontal asymptotes of the function

$$R(x) = \frac{x^4 + x^2 - 6}{2x^4 - 54x}.$$

Solution: Factor both numerator and denominator, then apply the asymptote theorem to get $y = 1/2$ as the horizontal asymptote, and $x = 0, x = 3$ for vertical asymptotes.

2. Solve the equation $3(2x - 5(x - 1) + 7x) = 15$.

Solution: $x = 0$ is the only solution.

3. Solve $x - 3/x = 6$.

Solution: Use the quadratic formula on $x^2 - 6x - 3 = 0$ to get $x = \frac{6 \pm \sqrt{36+12}}{2} = 3 \pm 2\sqrt{3}$.

4. What is the smallest root of $2x^3 + x^2 - 7x = 0$?

Solution: Factor to get $x(2x^2 + x - 7) = 0$ and then use the quadratic formula on the quadratic to find the smallest root to be $\frac{-1-\sqrt{43}}{4}$.

5. Let

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 0 \\ x + 3 & \text{if } 0 \leq x \end{cases} \quad \text{and } g(x) = |x| - 4$$

Find a symbolic representation of $g \circ f(x)$.

6. Solve $-3 < 2x - 1 < 5$.

Solution: Add 1 to all three parts to get $-3 + 1 < 2x < 6$ from which it follows that $-1 < x$ and $x < 3$. Writing this in one, $-1 < x < 3$.

7. Find the vertex and sketch the graph of $y = x^2 + 2x + 4$.

Solution: Complete the square to get $y = (x + 1)^2 + 3$ which has vertex at $(-1, 3)$.

8. What is the domain of

$$f(x) = \frac{\sqrt{x^2 - 1}}{x + 4}?$$

9. Find the center and the radius of the circle given by

$$x^2 + 6x + y^2 - 4y = -4.$$

Solution: The center is $(-3, 2)$ and the radius is 3.

10. What is the midpoint of the line segment from $(3, -6)$ to $(5, 12)$?

Solution: use the formula to get $((3 + 5)/2, (-6 + 12)/2) = (4, 3)$.

11. Suppose g is defined by $g(x) = (4 - x)/6$. Let f be the inverse of the function g . Find a symbolic representation of f .

Solution: $y = -6x + 4$.