

April 2000

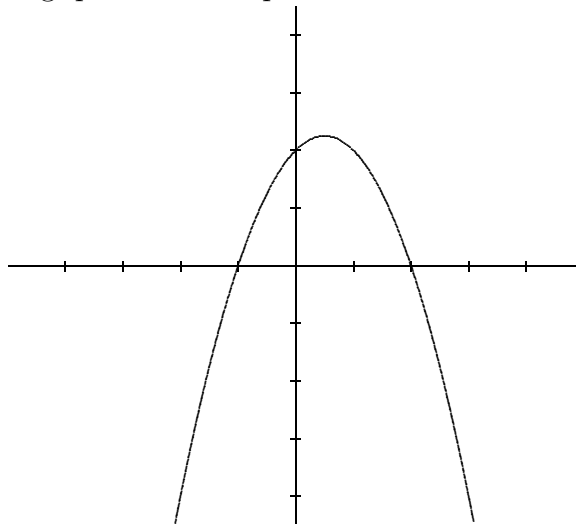
Your name \_\_\_\_\_

Problems count 5 points each.

1. What is the (implied) domain of the function  $h(x) = (x+1)^2 \cdot x \cdot \sqrt{4-x^2}$ ? Also, how many zeros does it have?

**Solution:** Domain is  $[-2, 2]$  and there are four distinct zeros,  $x = -2$ ,  $x = -1$ ,  $x = 0$ , and  $x = 2$ .

2. The graph of  $f(x) = ax^2 + bx + c$  is shown for certain values of  $a$ ,  $b$ , and  $c$ . Which of the following quantities are positive? Circle all that apply.



- (A)  $a$     (B)  $c$     (C)  $b^2 - 4ac$     (D)  $9a + 3b + c$     (E)  $a + b + c$

**Solution:** Note that  $c = f(0) = 2$ , and  $D = b^2 - 4ac$  is positive because  $f$  has two zeros. Finally,  $f(1) = a + b + c > 2$ , so it too is positive.

3. Solve  $\sqrt{5+x} = 5\sqrt{x}$ .

**Solution:** Square both sides to get  $5+x = 25x$  or  $24x = 5$ , so  $x = 5/24$ .

4. Find an equation for the line perpendicular to  $2x - y = 3$  that goes through the point  $(1, 3)$ .

**Solution:** The slope is  $-1/2$ , so the line is  $y - 3 = -1/2(x - 1)$  or  $y = -x/2 + 7/2$ .

5. Solve  $x + 1 = 12/x$ .

**Solution:** Message the given equation to get  $x^2 + x - 12 = 0$  which factors to give the two roots  $x = 3$  and  $x = -4$ .

6. Find an  $x$  such that  $\log_{10}(6x^2 - 3) = 2$ .

**Solution:** Translate this to exponential notation to get  $10^2 = 6x^2 - 3$  and solve to get  $x = \sqrt{103/6} \approx \pm 4.14$ .

7. Solve the inequality

$$x^3 - x^2 \leq 12x - 2x^2.$$

**Solution:** The inequality is equivalent to

$$x^3 + x^2 - 12x \leq 0,$$

the left side of which factors nicely. Use the test-interval idea to get critical points  $-4, 0$  and  $3$ . Then test to find that the solution is  $(-\infty, -4] \cup [0, 3]$ .

8. What is the time required for a 6% investment compounded monthly to double? ....to triple?

**Solution:** Recall that  $A = P(1 + r/n)^{nt}$  where  $A$  is the amount,  $P$  the principle,  $r$  is the annual rate,  $t$  is the time in years, and  $n$  is the number of compoundings per year. Doubling means  $A = 2P$  so we need to solve

$$2 = (1 + 0.06/12)^{12t}$$

or  $2 = 1.005^{12t}$ . Take logs of both sides to get  $t = \ln 2 / 12 \ln 1.005 \approx 11.58$  years. The triple time is the solution to  $3 = 1.005^{12t}$ , so it takes  $t = 18.36$  years for such an investment to triple.

9. Solve  $\frac{1}{x-1} + \frac{x}{x+1} = 4$ .

**Solution:** Find a common denominator and add the fractions to get the quadratic  $3x^2 - 5 = 0$  which then leads to  $x = \pm \sqrt{5/3} \approx \pm 1.29$ .

10. Find

$$\log_4 \left( \frac{8x^3}{2y} \right)$$

in terms of  $x$  and  $y$ .

**Solution:**  $\log_4 \left( \frac{8x^3}{2y} \right) = \log_4 8 + 3 \log_4 x - \log_4 2 - \log_4 y = \log_4 4 + \log_4 2 - \log_4 2 + 3 \log_4 x - \log_4 y = 1 + 3 \log_4 x - \log_4 y$ .