November 15, 2001 Your name ____________________

The first 6 problems count 7 points each and the final ones counts as marked. Problems 1 through 6 are multiple choice. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on multiple choice items. You must show your work on the other problems. The total number of points available is 124.

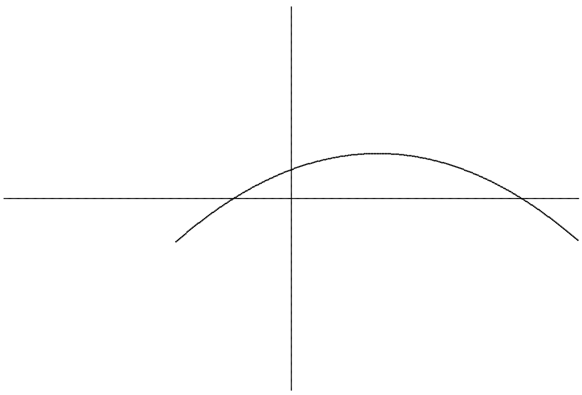
1. What is the remainder when $x^3 - 4x^2 + 5x - 6$ is divided by $x - 2$?

   (A) -4 **(B)** -2 **(C)** 0 **(D)** 2 **(E)** 4

   Solution: A. By the remainder theorem, the remainder when $p(x)$ is divided by $x-2$ is just $p(2)$, which is $p(2) = 2^3 - 4\cdot 2^2 + 5\cdot 2 - 6 = 8 - 16 + 10 - 6 = -4$.

2. The function f shown below is a quadratic polynomial with zeros $x = -1$ and $x = 4$ and a y-intercept 1. What is the value of $f(2)$?

   (A) 1 **(B)** $3/2$ **(C)** 2 **(D)** $5/2$ **(E)** 3



   Solution: B. The function must be of the form $f(x) = a(x+1)(x-4)$. In order that $f(0) = 1$, we must have $a(1)(-4) = 1$ which implies that $a = -1/4$. In this case $f(2) = (-1/4)(2+1)(2-4) = 3/2$.

3. What is the degree of the polynomial

$$(x^2 + 1)^4 (x + 2)^3?$$

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

   Solution: D. The degree is just the value of $2 \cdot 4 + 3 = 11$.

4. When the polynomial $(2x+1)^2(3x+2)^2(4x+3)^2$ is written in standard form, what is the coefficient a_6 of the x^6 term?

   (A) 36 **(B)** 144 **(C)** 288 **(D)** 576 **(E)** 1024

   Solution: D. The coefficient of x^6 is just the product $2^2 \cdot 3^2 \cdot 4^2 = 576$.

5. Referring again to the same polynomial, what is the constant term a_0?

   (A) 1 **(B)** 4 **(C)** 9 **(D)** 16 **(E)** 36

   Solution: E. The constant term is the product $1^2 \cdot 2^2 \cdot 3^2 = 36$.

6. What is the sum of the x-intercepts of

$$(x-2)^3(2x+7)^4 - (x-2)^4(2x+7)^3 = 0?$$

   (A) $-21/2$ **(B)** $-3/2$ **(C)** $-1/2$ **(D)** 0 **(E)** 9/2

   Solution: A. To determine the intercepts, factor the polynomial as follows: $(x-2)^3(2x+7)^3(2x+7-(x-2)) = (x-2)^3(2x+7)^3(x+9)$ which has three zeros, $x=2, x=-7/2$, and $x=-9$. Their sum is $-21/2$.

   On all the following questions, **show your work.**

7. (12 points) Find five simple functions a, b, c, d, and e such that the composite function $a \circ b \circ c \circ d \circ e(x) = \sqrt{3x^2-6}+7$.

   Solution: The five functions are $a(x) = x+7, b(x) = \sqrt{x}, c(x) = x-6, d(x) = 3x$, and $e(x) = x^2$.

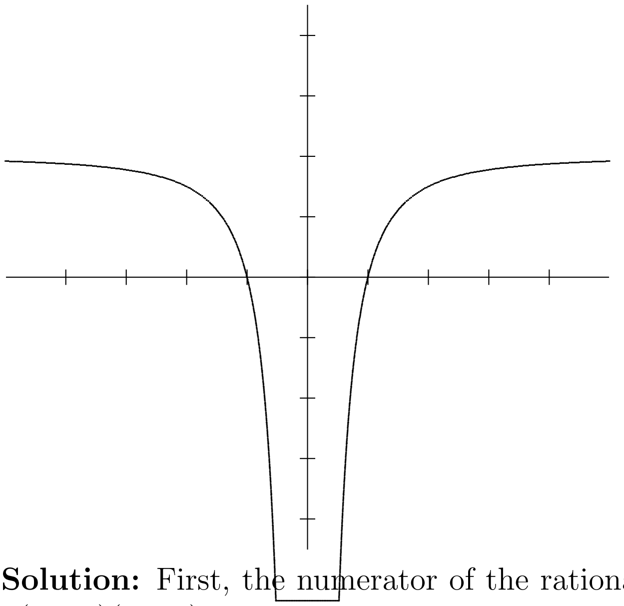
8. (15 points) Find the vertex of the parabola $f(x) = 3x^2-6x+21$ by completing the square. Explain why $f(x)$ is smaller at its vertex than any other point of its domain.

   Solution: Write $f(x) = 3(x^2-2x+7)$, and complete the square. This gives $f(x) = 3(x^2-2x+1-1+7) = 3((x-1)^2+6)$. Multiplying the 3 back in gives $f(x) = 3(x-1)^2+18$ which is of the right form. So we can read off the vertex (h,k) as $(1,18)$. Notice that $f(1) = 3(1-1)^2+18$ and that $f(x)$ is larger than 18 whenever x is not 1 because the $(x-1)^2$ term makes a positive contribution to the value of $f(x)$.

9. (20 points) Find the x-intercepts of the function $f(x) = x^3 - 7x^2 + 11x - 5$.

   Solution: Try evaluating at the possible rational roots, $\pm 1, \pm 5$. Note that $f(1) = 1 - 7 + 11 - 5 = 0$, so $x - 1$ is a factor. Divide the function by $x - 1$ to get $f(x) = (x - 1)(x^2 - 6x + 5) = (x - 1)(x - 1)(x - 5)$.

10. (15 points) The rational function $r(x)$ has asymptotes $y = 2$ and $x = 0$. It has x-intercepts at ± 1. Find a symbolic representation of $r(x)$.



   Solution: First, the numerator of the rational function must be of the form $a(x-1)(x+1)$ in order to make r have zeros ± 1. Also, the denominator must have the same degree and must have 0 as a zero. Thus $r(x) = \frac{a(x^2-1)}{x^2}$. Next a must be such that $y = 2$ is the horizontal asymptote. This happens when $a = 2$. This $r(x) = \frac{2x^2-2}{x^2}$.

11. (20 points) Suppose the demand equation for WIDGETS is given by $x = -5p + 100, \quad 0 \leq p \leq 20$ where p is the price in dollars and x is the number of Widgets sold.

   (a) Solve the demand equation for p and use the result to express the revenue function R as a function of x.

   Solution: We can easily solve for p to get $p = (-1/5)(x - 100)$. Then $R(x) = p \cdot x = (-1/5)(x - 100)x = (100x - x^2)/5 = -x^2/5 + 20x$.

   (b) What is the revenue if 15 units are sold?

   Solution: $R(15) = -15^2/5 + 20 \cdot 15 = -45 + 300 = 255$.

   (c) Find the vertex of $R(x)$.

   Solution: Complete the square to get $R(x) = (-1/5)(x^2 - 100x + 2500 - 2500) = (-1/5)(x - 50)^2 + 500$. Thus, $R(x) = (-1/5)(x - 50)^2 + 500)$. Hence the vertex is $(h, k) = (50, 500)$.