November 15, 2002 Your name ____________________________

The first 3 problems count 7 points each and the final ones counts as marked. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on multiple choice items. You must show your work on the other problems. The total number of points available is 111.

1. The rational function f is defined by

$$f(x) = \frac{(x-4)(x-2)}{(x^2-4)(x-3)}.$$

Which of the following lines is **not** an asymptote?

(A) $y = 0$ **(B)** $x = 3$ **(C)** $x = -2$ **(D)** $x = 2$ **(E)** $x = 4$

2. Which line are asymptotes of

$$f(x) = \frac{x^2(x^2-4)}{x^2(x-3)(x+2)}.$$

Circle all that apply.

(A) $y = 0$ **(B)** $y = 1$ **(C)** $x = 3$ **(D)** $x = -2$ **(E)** $x = 2$

3. When $\dfrac{(2x+2)(x-1)^2}{x(x-3)(x+4)}$ is expressed in rational function form

$$\frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0}{b_m x^m + b_{m-1}x^{m-1} + \ldots + b_0},$$

what is the value of $\frac{a_n}{b_m} + a_0 + b_1$?

(A) 1 **(B)** 5 **(C)** 9 **(D)** 11 **(E)** 12

4. (30 points) The rational function f is defined by

$$f(x) = \frac{(x-4)^2(3x-2)^3}{(x^2-4)(2x-7)(x+1)^2}.$$

Do a complete analysis of the function discussing asymptotes of both types and intercepts. Sketch the graph on the axes provided. Notice that the numerator, but not the denominator is in factored form.

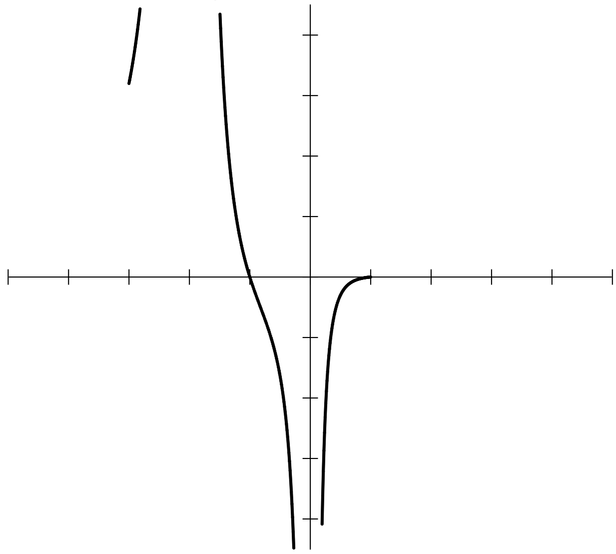
(a) What is the domain of f?

(b) What are the x-intercepts?

(c) What are the vertical asymptotes?

(d) Discuss the horizontal asymptote(s)?

5. (30 points) The graph on the grid provided below satisfies all the following. It has zeros at $x = 1$ and $x = -1$, vertical asymptotes $x = -2$ and $x = 0$ and a horizontal asymptote $y = 2$. Notice that both sides of the function go down at the $x = 0$ asymptote and both go up at the $x = -2$ asymptote.



Find a symbolic representation of such a function.

6. (30 points) Find all the zeros of the polynomial $p(x) = 2x^4 - x^3 - 20x^2 + 13x + 30$. A calculator solution to this problem is not acceptable. You must find the roots using algebra, use division to simplify the problem (ie, find the depressed polynomial), then find the zeros of the depressed polynomial repeatedly.