

December 4, 2000

Your name _____

The first 7 problems count 7 points each and the final ones count as marked. You do not need to show your work on problems 1 through 7. There are 131 points available on this test.

1. Combine into a single logarithm: $\ln(x + 1) + 3 \ln(x) - 2 \ln(x - 2)$

(A) $\frac{\ln(3x^2 + 3x)}{\ln(2x - 4)}$ (B) $\ln\left(\frac{x^4 + x^3}{(x - 2)^2}\right)$ (C) $\ln\left(\frac{x^4 + x^3}{(2x - 4)^2}\right)$

(D) $\ln(2x + 5)$ (E) $\frac{3\ln(x^2 + x)}{2\ln(x - 2)}$

2. Which of the following equations represents a parabola with vertex at the point $(2, 5)$ which passes through the point $(0, 8)$?

(A) $f(x) = x^2 - 2x + 8$ (B) $f(x) = x^2 - 4x + 8$ (C) $f(x) = \frac{3}{4}(x - 2)^2 + 5$
(D) $f(x) = (x - 2)^2 + 5$ (E) $f(x) = (x + 2)^2 + 5$

3. Consider the function defined by

$$h(x) = \frac{(x^2 + 5x - 6)(x^3 - 8)(x^2 - 2x + 4)}{(x^3 + 8)(3x - 6)(x - 6)(x + 2)^2}.$$

Circle all the equations of vertical asymptotes

(A) $x = -6$ (B) $x = -2$ (C) $x = 0$ (D) $x = 2$ (E) $x = 6$

4. For the same function h as in problem 3., circle all the equations of horizontal asymptotes.

(A) $y = 0$ (B) $y = 1/3$ (C) $y = 1/2$ (D) $y = 2/3$ (E) $y = 1$

5. Solve: $10 = e^{5x}$

(A) $x = \ln(2)$ (B) $x = \frac{\ln(10)}{5}$ (C) $x = \frac{\ln(10)}{\ln(5)}$

(D) $x = \ln(5)$ (E) $x = \frac{\ln(10)}{e^5}$

6. Solve: $\log(3x + 2) = 2\log(6)$

(A) $x = \frac{38}{3}$ (B) $x = \frac{34}{3}$ (C) $x = \frac{14}{3}$ (D) $x = \frac{10}{3}$ (E) $x = \frac{4}{3}$

7. If \$3,000 is invested in an account which pays 7.5% compounded continuously, what will the balance be at the end of 10 years?

(A) \$5250.00 (B) \$5,254.12 (C) \$6,183.09

(D) \$6,351.00 (E) \$22,500.00

Free response section. Show your work. No partial credit will be given without clear evidence of a correct method.

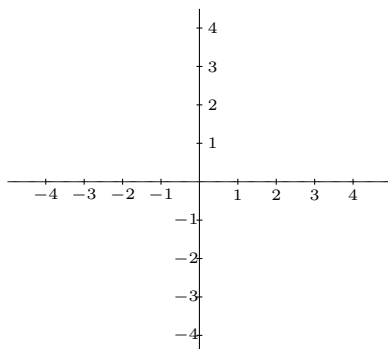
8. (20 points) Consider the polynomial function $p(x) = x^3 - 5x^2 + \frac{9}{4}x + 7$.

(a) Compute the values $p(0), p(1), p(2), p(3)$, and $p(4)$.

(b) One of the values above is 0. Therefore you can use the factor theorem to write $p(x)$ in the form $(x - r)(ax^2 + bx + c)$, where r is the number in part a for which $p(r) = 0$. Use long division (or synthetic division) to find the values of a, b and c .

(c) Find the other two zeros of p using the quadratic formula.

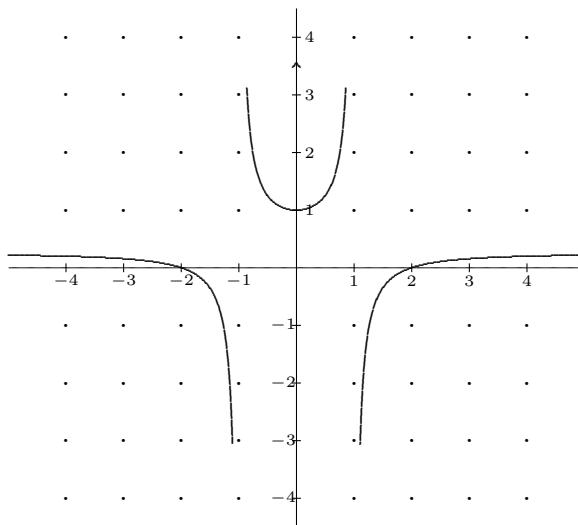
(d) Sketch the graph of p on the grid provided.



9. (10 points) Find the inverse of the function defined by $f(x) = \frac{2x+3}{4}$. Then prove that your answer $g(x)$ is correct by computing both composite functions $f \circ g(x)$ and $g \circ f(x)$.
10. (12 points) Suppose that \$800 is deposited into an account with an annual percentage rate of 6%.
- (a) What is the balance in the account after 3 years, assuming that compounding takes place quarterly? Round your answer to the nearest penny.
- (b) How many years (to the nearest hundredth of a year) does it take the account to double, again compounding quarterly?

- (c) How many years (to the nearest hundredth of a year) does it take the account to double, this time compounding continuously?

11. (20 points) Asymptotes and graphs. The graph of a function with zeros at $x = -2$ and $x = 2$ and vertical asymptotes $x = -1$ and $x = 1$ and a horizontal asymptote $y = 1/4$ is shown below. Find a symbolic representation of such a function.



12. (20 points) Explain how you can describe the graph of the quadratic equation

$$y = ax^2 + bx + c$$

based on the coefficients a , b , and c . Hint: it may be useful to define the discriminant D to be $b^2 - 4ac$. In particular, address the questions (a) does the curve open upwards or downwards, and (b) does it have x -intercepts?