

Quiz 5

Let $f(x) = x^2 + 1$. Compute the derivative f' of f using the definition of derivative.

Solution: We need to find

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

So first find the difference quotient, $\frac{f(x+h)-f(x)}{h}$.

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(x+h)^2 + 1 - (x^2 + 1)}{h} \\ &= \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h} \\ &= \frac{2xh + h^2}{h} \\ &= \frac{h(2x + h)}{h} \\ &= 2x + h \end{aligned}$$

Next, take the limit as h approaches 0 to get $2x$.