

Use the definition of derivative to find $f'(a)$ for the function $f(x) = 3x - x^2$. Use this information to find the slope of the line tangent to the graph of f at the point $(-2, -10)$. Then find an equation for the tangent line.

Solution: Compute the difference quotient

$$\begin{aligned}
 f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} && = \\
 &= \lim_{h \rightarrow 0} \frac{3(a+h) - (a+h)^2 - (3a - a^2)}{h} && = \\
 &= \lim_{h \rightarrow 0} \frac{3a + 3h - (a^2 + 2ah + h^2) - 3a + a^2}{h} && = \\
 &= \lim_{h \rightarrow 0} \frac{3h - a^2 - 2ah - h^2 + a^2}{h} && = \\
 &= \lim_{h \rightarrow 0} \frac{3h - 2ah - h^2}{h} && = \\
 &= \lim_{h \rightarrow 0} \frac{h(3 - 2a - h)}{h} && = \\
 &= \lim_{h \rightarrow 0} 3 - 2a - h = 3 - 2a
 \end{aligned}$$

Now when $a = -2$, we have $f'(-2) = 3 - 2(-2) = 7$, so the line in question is given by $y - (-10) = 7(x - (-2))$, which in simplified form is $y = 7x + 4$.