

Quiz 8

Find the intervals of concavity of the function

$$g(x) = (x^2 - 6)^2,$$

and classify each such interval.

Solution: Compute the first derivative using the chain rule. $g'(x) = 2(x^2 - 6) \cdot 2x = 4x(x^2 - 6)$, and then the second derivative by the product rule: $g''(x) = 4(x^2 - 6) + 2x \cdot 4x = 12x^2 - 24$. Therefore, $g''(x) = 0$ has two solutions, $x = \sqrt{2}$ and $x = -\sqrt{2}$. Next test the sign of the second derivative on each of the three intervals determined by the two 'critical' numbers. You see that $g(x)$ is concave up on $(-\infty, -\sqrt{2})$ and on $(\sqrt{2}, \infty)$ and concave down on the interval $(-\sqrt{2}, \sqrt{2})$.