1. Ashley noticed that the set of ages of her relatives, all of whom were whole numbers in the range 1 up to 100 inclusive, has the unusual property that no two of them multiplied together is a perfect square. What is the largest number of relatives Ashley could have?

2. Some properties of relations. We describe below some important properties that relations might or might not have. A relation R on a set A is called

   R. Reflexive if $\forall x \in A, xRx$.

   S. Symmetric if $\forall x, y \in A, xRy \Rightarrow yRx$.

   A. Antisymmetric if $\forall x, y \in A, xRy$ and $yRx \Rightarrow x = y$.

   T. Transitive if $\forall x, y, z \in A, xRy$ and $yRz \Rightarrow xRz$.

   Now let $A = \{1, 2, 3\}$. There are $2^4 = 16$ subsets of $\{R, S, A, T\}$. Find a relation on A for each of these subsets. For example, consider the subset $\{S, A, T\}$. We seek to find a relation on $\{1, 2, 3\}$ that is symmetric, antisymmetric, and transitive, and *not* reflexive. To keep the relation from being reflexive, we must exclude one of the three ordered pairs $(1, 1), (2, 2), (3, 3)$. However, two of these could be included. So lets try $H = \{(1, 1), (2, 2)\}$. Is this symmetric? Is it transitive? Is it antisymmetric? Sketch the digraph of the relation and notice that it has just two loops. After some thought, you'll decide that H is symmetric, antisymmetric, and transitive. There are 15 other subsets of $\{R, S, A, T\}$. Find a relation for each of these, or prove that certain combinations do not exist.