

December 11, 2002

Your name _____

It is important that you **show your work**. The total value of this test is 190 points.

1. (20 points) Find a relation R on the set $\{1, 2, 3, 4\}$ satisfying each set of requirements. Leave your answer in matrix form.
 - (a) Reflexive, antisymmetric and symmetric.
 - (b) Not symmetric and not antisymmetric.

2. (20 points) Prove that

$$1 + 3 + 3^2 + \dots + 3^n = \frac{3^{n+1} - 1}{2}$$

for $n = 0, 1, 2, \dots$

3. (15 points) Use the Euclidean algorithm to solve the decanting problem for decanters of sizes 217 and 975. In other words, find integers x and y such that $\gcd(217, 975) = 217x + 975y$.
4. (10 points) Find the remainders when each N is divided by d .
- (a) $N = 5^{41}$ and $d = 7$
 - (b) $N = 123,456,789,101,112$ and $d = 11$

5. (20 points) Find the base 4 and base -4 representation of each of the numbers below.
- (a) 217
 - (b) 36.75
6. (15 points) Suppose $A, B,$ and C are three sets that satisfy $|ABC| = |ABC\bar{C}| = |A\bar{B}C| = |\bar{A}BC| = |A\bar{B}\bar{C}|$ and $|A| = 20, |B| = 17,$ and $|C| = 19.$ What is $|A \cup B \cup C|$? Recall that UV refers to $U \cap V.$
7. (20 points)
- (a) Prove that the intersection of two symmetric relations on the set A is also symmetric.
 - (b) Prove that every subset of an antisymmetric relation is antisymmetric.

8. (20 points) How many relations R on the set $S = \{a, b, c\}$ are there such that
- (a) R is not reflexive?
 - (b) R is antisymmetric?
 - (c) R is an equivalence relation.
 - (d) R is reflexive, antisymmetric and transitive. (very hard!)

9. (20 points) Let $S = \{1, 2, 3, 4, 5, 6\}$.

- (a) How many four-digit numbers can be made using the members of S as digits if repetition of digits is allowed.
- (b) How many four-digit numbers can be made using the members of S as digits if repetition of digits is not allowed.
- (c) How many four-digit multiples of 15 can be made using the members of S as digits if repetition of digits is allowed.
- (d) How many four-digit multiples of 6 can be made using the members of S as digits if repetition of digits is not allowed.

10. (15 points) Let $S = \{a, b\}$ and let $R = \{(a, a), (a, b), (b, a)\}$.
- (a) Prove that R is not antisymmetric.
 - (b) Prove that R is symmetric.
 - (c) Prove that R is not transitive.
11. (15 points) Let Z denote the set of all integers. Define the relation R on Z by xRy if $x - y$ is even.
- (a) Prove that R is reflexive.
 - (b) Prove that R is symmetric.
 - (c) Prove that R is not antisymmetric.
 - (d) Prove that R is transitive.
 - (e) Find the cells $[0]$ and $[1]$.