

February 14, 2000

Your name _____

It is important that you **show your work**. The total value of this test is 110 points.

1. (10 points) Find the base -6 representation of 29.

2. (20 points)

(a) Use the division algorithm to find the unique integers r and q satisfying

$$297 = 73q + r \text{ and } 0 \leq r < 73.$$

(b) Solve the decanting problem for containers of sizes 73 and 297; that is find integers x and y satisfying

$$73x + 297y = d$$

where d is the GCD of 73 and 297.

3. (10 points) Find the base 6 representation of each of the following:

(a) 247

(b) $8\frac{13}{36}$

(c) 0.15

4. (15 points)

(a) Construct the base 6 addition table and the base 6 multiplication table.

(b) Use the tables in (a) to carry out the multiplication $1051_6 \times 204_6$.

(c) Convert the three numbers 1051_6 , 204_6 , and your answer in (b) to their decimal equivalents and carry out the multiplication in decimal representation to check your answer to (b).

5. (20 points) Notice that

$$2 = 2 = 2 \cdot 1 \quad (1)$$

$$2 + 4 = 6 = 3 \cdot 2 \quad (2)$$

$$2 + 4 + 6 = 12 = 4 \cdot 3 \quad (3)$$

$$2 + 4 + 6 + 8 = 20 = 5 \cdot 4 \quad (4)$$

- (a) List the next three equations suggested by the pattern.
- (b) Given that the four equations above are the 1st, 2nd, 3rd, and 4th, write the n^{th} equation of the sequence.
- (c) Use mathematical induction to prove that the n^{th} equation is true for all positive integer values of n .

6. (15 points) Divisors and Prime factorization

(a) Find two different numbers a and b both of which are multiples of 6 such that each one has exactly 8 positive integer divisors.

(b) Find the greatest common divisor (GCD) of your a and b .

(c) Find the least common multiple (LCM) of your a and b .

7. (20 points) Compute the remainders when each n below is divided by the given d .

(a) $n = 3^{2001}$, $d = 10$

(b) $n = 5^{2001}$, $d = 7$

(c) $n = 123,456,789,012,345,678$, $d = 9$

(d) $n = 123,456,789,012,345,678$, $d = 11$