February 14, 2001

Your name ______________________________

It is important that you **show your work**. There are 125 points available on this test.

1. (15 points) Find a pair of integers m and n such that m/n is reduced and $m/n = 21.3\overline{64}$.

2. (20 points)

   (a) Find the base 6 representation of 129.

   (b) Find the base -6 representation of 129.

   (c) Find the base 2 representation of 6.125.

3. (20 points)

   (a) Use the division algorithm to find the unique integers r and q satisfying

   $$377 = 39q + r \text{ and } 0 \leq r < 39.$$

   (b) Solve the decanting problem for containers of sizes 377 and 39; that is find integers x and y satisfying

   $$377x + 39y = d$$

   where d is the GCD of 39 and 377. containers of sizes 387 and 39; that is

4. (20 points) Notice that

$$\begin{aligned} 1 &= 1 = 1^2 & (1)\\ 1+3 &= 4 = 2^2 & (2)\\ 1+3+5 &= 9 = 3^2 & (3)\\ 1+3+5+7 &= 16 = 4^2 & (4) \end{aligned}$$

(a) List the next three equations suggested by the pattern.

(b) Given that the four equations above are the $1^{st}, 2^{nd}, 3^{rd}$, and 4^{th}, write the n^{th} equation of the sequence. Notice that in the 4^{th} equation, the last summand is 7 (not 4).

(c) Use mathematical induction to prove that the n^{th} equation is true for all positive integer values of n.

5. (15 points) Divisors Let p, q, and r be three different prime numbers. In terms of p, q, and r, compute
   (a) $\text{GCD}(p^3q^2r, p^2qr^3)$
   (b) $\text{LCM}(p^3q^2r, p^2qr^3)$
   (c) the number of divisors of p^3q^2r.

6. (20 points) State the Fundamental Theorem of Arithmetic. Then use it to give an argument that the square root of 2 is irrational. Why is it not possible to prove that $\sqrt{4}$ is not rational using this method? Elaborate.

7. (15 points) Prove that for any integer $n \geq 5$, $2^n > n^2$.