

1. (15 points) Solve the decanting problem for containers of sizes 180 and 266; that is, find integers x and y satisfying

$$180x + 266y = d$$

where d is the GCD of 180 and 266.

2. (12 points) Let $N = 17017$.

(a) What is the prime factorization of N ?

(b) What is the remainder when N is divided by 9?

(c) What is the remainder when N is divided by 11?

(d) How many (positive integer) divisors does N have?

3. (10 points) Find a pair of relatively prime integers m and n such that $\frac{m}{n} = 0.\overline{857142}$.

4. (10 points) Find the base 5 representation of the number 717 in two ways.

5. (10 points) Find the base 5 representation of the number $2/3$.

6. (20 points) Notice that

$$1 = 1 \tag{1}$$

$$1 + 3 = 4 \tag{2}$$

$$1 + 3 + 5 = 9 \tag{3}$$

$$1 + 3 + 5 + 7 = 16 \tag{4}$$

- (a) List the next three equations suggested by the pattern.
- (b) Given that the four equations above are the 1st, 2nd, 3rd, and 4th, write the n^{th} equation of the sequence.
- (c) Use mathematical induction to prove that the n^{th} equation is true for all positive integer values of n .

7. (10 points) You are playing a game of Bouton's Nim, and the position is $(17, 15, 13, 9, 6)$. Is this a safe position? If not, find the winning move.
8. (8 points) What is the remainder when 3^{2002} is divided by 5?
9. (10 points) Find digits a and b for which the number $11ab3$ is a multiple of 99.
10. (15 points)
- (a) Suppose $p, q,$ and r are different prime numbers. How many (positive integer) divisors does $N = p^2q^3r^5$ have?
 - (b) Find the greatest common divisor (GCD) of N and $M = pq^4$.
 - (c) Find the least common multiple (LCM) of N and $M = pq^4$.
11. (10 points) Let $x = \log_2 5$. This means that $2^x = 5$. If x is rational, then there are integers m and n such that $2^{m/n} = 5$, or equivalently, $2^m = 5^n$. Explain in detail why $\log_2 5$ is irrational.