

September 23, 2002

Your name _____

1. (10 points) Find the base 6 representation of 2002.
2. (10 points) Find the base 6 representation of $1/7$.
3. (10 points) Find a pair of relatively prime integers m and n for which $\frac{m}{n} = 1.2\bar{5}$.
Two numbers are relatively prime if their greatest common divisor is 1.
4. (10 points) Find a base 6 digit d such that $22d1_6 = 6d4_9$.

5. (10 points) Find the best (winning) move in the game of Bouton's Nim $(14, 13, 12, 9)$.

6. (12 points) Let $M = 123,123$ and let $N = 194,040$.

(a) Find the prime factorizations of M and N .

(b) Compute $LCM(M, N)$

(c) Compute $GCD(M, N)$

(d) Find the number of divisors of N .

7. (15 points) Solve the decanting problem for containers of sizes 374 and 39; that is find integers x and y satisfying

$$374x + 39y = d$$

where d is the GCD of 39 and 374.

8. (10 points) Find the representations of the integers 1 through 13 in base -4 .

9. (20 points) Look at the four equations below.

$$\begin{aligned}2 &= 2 \cdot 1 \\2 + 4 &= 3 \cdot 2 \\2 + 4 + 6 &= 4 \cdot 3 \\2 + 4 + 6 + 8 &= 5 \cdot 4\end{aligned}$$

a. Write the next three equations in the sequence.

b. If the four equations above correspond to $k = 1, 2, 3,$ and $4,$ what is the n^{th} equation?

c. Prove by mathematical induction that the n^{th} equation is true for all integers $n \geq 1.$