

5. (10 points) Find a digit d such that $d111_6 = 1d46_7$.

6. (12 points) Let $M = 324,324$ and let $N = 194,040$.

(a) Find the prime factorizations of M and N .

(b) Compute $LCM(M, N)$

(c) Compute $GCD(M, N)$

(d) Find the number of divisors of M .

7. (15 points) Solve the decanting problem for containers of sizes 99 and 79; that is find integers x and y satisfying

$$99x + 79y = d$$

where d is the GCD of 99 and 79.

8. (15 points) Find the remainder in each case below.

(a) When $N = 5^{2003}$ is divided by 7.

(b) When $N = 123444555666777888999$ is divided by 9.

(c) When $N = 123444555666777888999$ is divided by 11.

9. (10 points) Find the best (winning) move in the game of Bouton's Nim (27, 23, 22, 19).

10. (20 points) Look at the four equations below.

$$\begin{aligned}1 \cdot 1! &= 2! - 1 \\1 \cdot 1! + 2 \cdot 2! &= 3! - 1 \\1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! &= 4! - 1 \\1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4! &= 5! - 1\end{aligned}$$

(a) Write the next three equations in the sequence.

(b) If the four equations above correspond to $k = 1, 2, 3,$ and $4,$ what is the n^{th} equation?

(c) Prove by mathematical induction that the n^{th} equation is true for all integers $n \geq 1.$