Math 1165 **Test 2**

November 13, 2002 Your name ____________________________

There are a total of 150 points available on this test.

1. (20 points) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. For each property below find the number of subsets T of S that have the property.

   (a) T has exactly three elements.

   (b) $|T| = 5$ and T has three odd members and two even members.

   (c) T has no prime number members. Recall that 1 is not prime.

   (d) T has at least two odd and at least three even members.

2. (20 points) Again let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. For each property below find the number of five digit numbers that can be constructed using the members of S as digits and that have the property.

   (a) The number has a value of at least 20000.

   (b) The number has a value of at least 20000 and the digits are all different.

   (c) The number is a multiple of nine and the digits are all different.

   (d) The digits of the number are in increasing order from left to right. For example, 13457.

3. (20 points) Let Z denote the set of all integers, $Z = \{0, \pm 1, \pm 2, \ldots, \}$, and let Z^+ denote the set of positive integers.

   (a) Find a <u>one-to-one</u> function g from Z^+ <u>onto</u> Z.

   (b) Prove that your function is one-to-one.

   (c) Prove that your function is onto.

4. (20 points) Let A and B be sets with characteristic functions f_A and f_B. Compute the following characteristic functions of the terms of f_A and f_B.

   (a) $f_{A\cap B}$

   (b) $f_{A\cup B}$

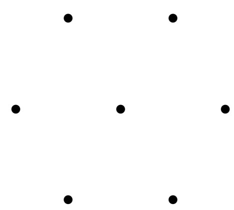
   (c) $f_{\overline{A}}$

   (d) Use the properties (a),(b), and (c) to compute the characteristic function of $A\cup(B\cup C)$. How does this anticipate the inclusion-exclusion principle for three sets?

5. (10 points) Suppose $f(n) = 5f(\frac{n}{2}) + n + 1$ and $f(4) = 40$. Find $f(1)$.

6. (15 points) Suppose one hundred students are polled about the academic preferences. Let A, B, and C denote the student who enjoy studying anthropology, biology, and chemistry respectively. For convenience, write AB to mean $A \cap B$, etc. Recall that $|X|$ denotes the number of elements of the set X. You are given the following information about the hundred students: $|ABC| = 2, |AB| = 7, |A \cup C| = 39, |\overline{A}BC| = 6, |A| = 25, |B| = 30,$ and $|C| = 20$. How many students do not enjoy any of the three types of courses?

7. (10 points) The points shown are the vertices of a regular hexagon with side length 1 together with the center of the hexagon. How many circles of radius 1 in the same plane have at least two vertices in the set?



8. (15 points) Find the number of integers from 1 to 600 inclusive that are not divisible by any of the numbers $3, 5$, and 7

9. (10 points) Solve: $a_n = 3a_{n-1} + 10a_{n-2}$ for $n \geq 2$ with the initial values $a_0 = 0$, $a_1 = 1$. Use your solution to find the value of a_{10}.

10. (10 points) Suppose $\mathcal{U}$ is a 14 element set with subsets A, B, and C satisfying $|A| = |B| = |C| = 7$, $|AB| = 4$, $|BC| = 3$, $|AC| = 2$, and $|ABC| = 1$. Compute the following cardinalities.

    (a) $|A \cup (B \cap \overline{C})|$

    (b) $|(A \times A) \cup (B \times B) \cup (C \times C)|$

    (c) $|(A \cap B) \times (B \cap C) \times (A \cap C)|$

    (d) $|\overline{(A \cup B \cup C) \times (A \cup B \cup C)}|$

    (e) $|\overline{(A \cup B \cup C)} \times \overline{(A \cup B \cup C)}|$