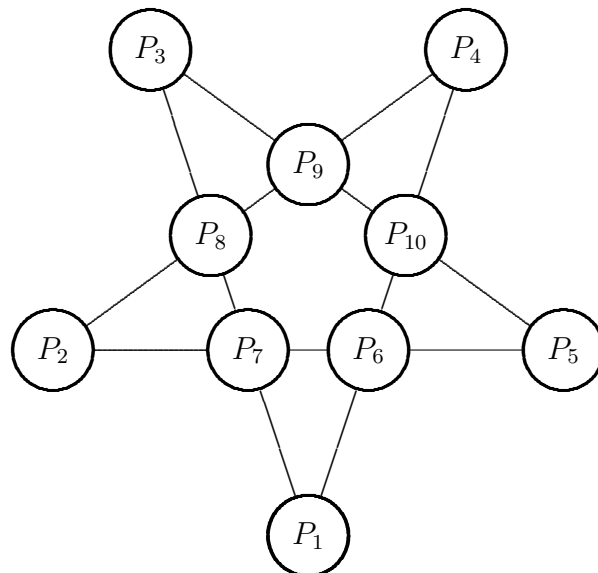


Symmetry

1. Find the volume of a rectangular box whose left side, front side, and bottom have areas of 10 square inches 15 square inches and 294 square inches, respectively.
2. Arrange the numbers 1, 2, 3, 4, 5, 6, 8, 9, 10, 12 in the ten locations so that the sum of the four numbers along each of the five lines is the same.



3. Now try the same problem with the numbers 1, 5, 7, 11, 18, 21, 24, 33, 42, 43.

Symmetry

4. Solve simultaneously:

$$x + 2y + z = 14$$

$$2x + y + z = 12$$

$$x + y + 2z = 18$$

5. Solve simultaneously:

$$x + y = 7$$

$$x + z = -2$$

$$y + z = 9$$

6. Solve simultaneously:

$$xy = -6$$

$$yz = -2$$

$$xz = 12$$

7. Solve simultaneously:

$$(x + 1)(y + 1) = 24$$

$$(y + 1)(z + 1) = 30$$

$$(x + 1)(z + 1) = 20$$

8. Solve simultaneously:

$$xy - x - y = 11$$

$$yz - y - z = 14$$

$$xz - x - z = 19$$

9. Solve simultaneously:

$$x(x + y + z) = 4$$

$$y(x + y + z) = 6$$

$$z(x + y + z) = 54$$

Symmetry

10. Solve simultaneously:

$$x + [y] + \langle z \rangle = 1.1$$

$$\langle x \rangle + y + [z] = 2.2$$

$$[x] + \langle y \rangle + z = 3.3$$

The notation $[x]$ is read ‘floor of x ’ and means the largest integer not greater than x . The notation $\langle x \rangle$ is read ‘fractional part of x ’ and means the $x - [x]$.

11. Given that a is a real number, solve simultaneously:

$$x^2 - xy = a$$

$$y^2 - xy = a(a - 1).$$

12. Solve simultaneously:

$$x_2 + x_3 + x_4 + \dots + x_n = 1$$

$$x_1 + x_3 + x_4 + \dots + x_n = 2$$

$$x_1 + x_2 + x_4 + \dots + x_n = 3$$

$$\vdots$$

$$x_1 + x_2 + x_3 + \dots + x_{n-1} = n.$$

13. A triangle has sides of lengths 13, 14, and 15. Its inscribed circle divides each side into two segments, making six altogether. Find the length of each segment.

14. Solve simultaneously:

$$xy + xz = 13$$

$$xz + yz = 25$$

$$xy + yz = 20.$$

Symmetry

15. Solve simultaneously:

$$\begin{aligned}2x_1 + x_2 + x_3 + x_4 + x_5 &= 6 \\x_1 + 2x_2 + x_3 + x_4 + x_5 &= 12 \\x_1 + x_2 + 2x_3 + x_4 + x_5 &= 24 \\x_1 + x_2 + x_3 + 2x_4 + x_5 &= 48 \\x_1 + x_2 + x_3 + x_4 + 2x_5 &= 96\end{aligned}$$

16. Solve the system of equations:

$$\frac{xyz}{x+y} = 7.2, \quad \frac{xyz}{y+z} = 4, \quad \frac{xyz}{x+z} = 4.5.$$

17. Solve the equation

$$\frac{x-3}{2001} + \frac{x-5}{1999} + \frac{x-7}{1997} + \frac{x-9}{1995} = \frac{x-2000}{4} + \frac{x-1998}{6} + \frac{x-1996}{8} + \frac{x-1994}{10}.$$

Advanced problems:

18. Suppose that for all positive x ,

$$11f(x+1) + 5f(1+1/x) = \log_{10} x.$$

Find $f(6) + f(33) + f(626)$.