

# Counting Problems

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1. Ten points are taken on the circumference of a circle. How many chords can be drawn by joining them in pairs in all possible ways? With these 10 points as vertices how many triangles can be drawn? How many convex hexagons?
2. In how many ways can 4 persons be selected from 5 married couples if
  - (a) (i) The selection must consist of 2 women and 2 men and (ii) a husband and wife cannot both be selected?
  - (b) There is no stipulation about married couples but the committee must still contain 2 men and 2 women?
3. In how many ways can 10 identical objects be placed in 4 different boxes if at least one object must go into each box? How many ways if one or more of the boxes may be left empty?
4. In how many ways can I buy a half dozen donuts if I can choose from among glazed, powdered and filled donuts and I can buy as many or few of any one kind as I wish?
5. In how many ways can a 6 volume encyclopedia be placed on a shelf. How many of these ways have NONE in the right position? How many of them have at least one in the right position? How many have exactly one in the right position?
6. How many distinguishably different 10-digit numbers can be obtained by arranging the digits 2233344455? How many distinguishably different 3 digit numbers could be formed from the same set?
7. A class consists of 12 girls and 10 boys. In how many ways can the class form a line if the girls always remain in order from shortest to tallest within the line and the boys always remain in order from tallest to shortest within the line?
8. In Square City, how many ways can one take a walk for 9 blocks if one always walks 5 blocks west and 4 blocks north?
9. How many distinguishable numbers greater than 3,000,000 can be formed from the digits 1, 1, 1, 2, 2, 3, 3?
10. If 5 people come to a table with labeled placecards but don't look at the cards before taking seats, in how many ways can they take seats so that no one is sitting by her proper placecard?

11. Twenty chairs are lined up in a row for the Princeton garlic-eating contest. Only five eaters attend the competition, but none will sit next to any other. In how many ways can the eaters be seated?
12. In how many ways can I get to my brothers home (traveling along east-west and north-south blocks) if he lives 9 blocks east and 7 blocks north of me? What if I want to stop first at my sister's house and she lives 2 blocks east and 6 blocks north of me?
13. What is the relationship between the following pairs of numbers?

$$\sqrt{5 + \sqrt{21}} + \sqrt{8 + \sqrt{55}} \quad \text{and} \quad \sqrt{6 + \sqrt{35}} + \sqrt{7 + \sqrt{33}}$$