

Prime Leaps, 1721

The history-cum-math teacher asked the class to name some years that they knew from history lessons. Johnny named 1066, the Battle of Hastings, and 1939, the outbreak of World War II.

The teacher then asked him to calculate the number of years between the two events, and Johnny correctly answered 873. Teacher then asked Johnny if that difference was a prime number and Johnny correctly answered that it was not prime since it is divisible by 3. The teacher then asked the class to find the longest list of years from 0, 1, 2, 3, 4, \dots , 1996 so that any two numbers in the list have a difference that is not prime. **What numbers are in the longest such list?**

Solution: The longest list can contain at most two from any eight consecutive numbers in 0, 1, 2, 3, 4, \dots . This is because if 0, say, is in the chosen list then 2, 3, 5, 7 are not. Then at most one of 1, 4, 6 can be in the list. So there is no hope of choosing 501 numbers all differing by non-primes (because the 3rd is at least 8, the fifth is at least 16, \dots , and 501st is at least 2000. We'll now show how to choose 500 numbers and we'll show that the choice is unique.

Clearly the list 0, 4, 8, 12, 16, \dots , 1996 contains 500 numbers and any two differ by a multiple of 4 (and so no difference is prime). Is there any other way of getting 500 numbers? Clearly we choose 0 first. Then the third choice must be at most 8 (because we cannot fit 498 choices in 9, 10, \dots , 1996). Similarly, the fifth choice must be at most 16, etc. etc.

So we start by choosing 0 (and the next must then be 1, 4, or 6). If we have chosen 1 also, then 2 through 8 must be rules out, but that means the third is greater than 8. So we cannot choose 1 as the second. If we choose 6 next, then 7 and 8 are rules out. In these cases, the third number is greater than 8. Thus the second number must be 4. Now we can start the argument again to deduce that the third choice must be 8, etc. etc.

So the answer is 0, 4, 8, 12, \dots , 1996- ie. all the years divisible by 4.